

Limit theorems for super-diffusions corresponding to the operator $Lu + \beta u - ku^2$

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Abstract

Consider a superdiffusion X corresponding to the operator $Lu + \beta u - ku^2$, where $\beta(x)$ is bounded from above and is in the Kato class, and $k(x) \geq 0$ is bounded on compact subset of \mathbf{R}^d . Let $-\Lambda$ be the L^∞ -spectral radius of the semigroup Q_t corresponding to the Schrodinger operator $Lu + \beta u$. We prove that if $\Lambda > 0$, the exponential growth rate of the total mass of X is Λ ; if $\Lambda < 0$, the exponential decay rate of the total mass of X is $\Lambda < 0$. We also describe the limiting behavior of $\exp(-\Lambda t)X_t(\mathbf{R}^d)$, where $X_t(\mathbf{R}^d)$ is the total mass of x at time t . In particular, in the case $\Lambda = 0$, under some restrict conditions on β , we give a sufficient and necessary condition for the superdiffusion X exhibiting weak extinction. It turns out that the branching rate function k affects the weak extinction, this should be compared with the known result that k does not affects the weak local extinction of X .

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