1. Suppose that the claims have a Gamma distribution, where the p.d.f. is given by $f_Y(x) = xe^{-x}, x > 0$. Assume that $\frac{\xi}{\lambda} = 4$. Find the adjustment coefficient $s^*$.  
Note: we need $M_Y(s^*) < \infty$.

2. Suppose that $c = 1$ and $\lambda = 2$. Assume that the ruin probability is given by

$$\psi(u) = 0.5e^{-0.7u} + 0.1e^{-1.2u} + 0.2e^{-2.8u}, \quad u \geq 0.$$  
(a) Find the adjustment coefficient $s^*$.  
(b) Find the mean of the claims $\mu$.  
(c) Find $M_Y'(s^*)$.  
(d) Is it true in general that $M_Y'(s^*) > \mu$? Why or why not?

3. Consider the so-called equilibrium probability density function for the claim sizes

$$g_Y(x) = \frac{1}{\mu} \int_x^\infty f_Y(y) \, dy \quad (x > 0).$$

Check that $g_Y(x)$ is indeed a probability density function. Prove that for any $s > 0,$

$$\int_0^\infty e^{sx} g_Y(x) \, dx = \frac{M_Y(s) - 1}{\mu s}.$$  
Conclude that $s^*$ is the adjustment coefficient if and only if

$$\int_0^\infty e^{sx} g_Y(x) \, dx = \frac{c}{\lambda \mu}.$$