MTH XXX Homework Template. **FirstName LastName**
Prof. M. Peszynska Due: Friday 02/13/1976

Problem 1. Discuss the error in the approximation of $f'(x)$ by the difference quotient $D_h f = \frac{f(x+h)-f(x)}{h}$.

Solution: We assume f is C^2 smooth in some interval $x \in [a,b]$ including $(x, x+h)$. We expand $f(x+h)$ with Taylor polynomial with the remainder in Lagrange form [1, p.203]

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(\xi) \;\; \xi \in (x, x+h). \tag{1}$$

We see then that $D_h f(x) = f'(x) + \frac{h}{2} f''(\xi)$.. Since f'' is bounded on $(x, x+h)$, the error $e(h) = \mid D_h f(x) - f'(x) \mid = O(h)$ is <u>of first order in h</u>.

Problem 2. Confirm the findings in Problem 1 with numerical experiments for $f(x) = \frac{1}{1+x^2}$.

Solution: We implement code (show the code if required). Figure 1 shows the convergence plot generated which confirms the first order of convergence. Table 1 shows again that the error $e(h)$ decreases linearly with h.

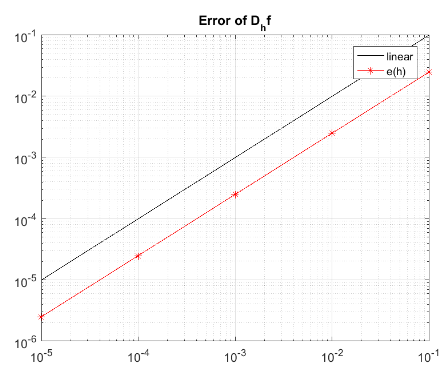

FIGURE 1. The error $e(h)$ appears to have the same slope as the linear trend.

h	$e(h)$	α
0.1	0.024886877828054	
0.01	0.002499876243744	0.998051906332859
0.001	0.000249999875134	0.999978717718038
0.0005	... include more data	

TABLE 1. Matching $e(h)$ to h^α reveals that $\alpha \approx 1$.

REFERENCES

[1] P.M.Fitzpatrick *Advanced Calculus*. American Mathematical Society, 2006.
[2] CTAN archive of the LaTeX packages `https://ctan.org/pkg/`