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HW 1

#13: (Direct proof)

Let $x \in S$ (S is nonempty hence such x exists).
S is bounded from above and below, therefore from Completeness Axiom there exists $\inf S$ and $\sup S$.
From definition of $\inf S$ and $\sup S$, we have

$$\inf S \le x$$

and $$x \le \sup S$$

Combining these two we get $\inf S \le x \le \sup S$
hence $\underline{\inf S \le \sup S}$.

(Contradiction).

Let $\inf S > \sup S$. Then for $x \in S$ we have (from def. of $\inf S$).

$$x \ge \inf S > \sup S \Rightarrow x > \sup S.$$

But from def. of $\sup S$ we must have $x \le \sup S$.
So we have a contradiction.