

SOLUTIONS

MTH 311/Peszynska Quiz 1, 9/28

Name _____

Student ID # _____

1. (2 pts)

For the sets below, find the following, if possible:

	two upper bounds	three lower bounds	sup S	inf S
$S = \{-1, 0\}$	1, 2	-1, -2, -100	0	-1
$S = \{1, 2, 4, 8, \dots\}$	1, 2	0, -1, -2	1	-1
$S = \{-1, 1, -2, 2, -3, 3, \dots\}$	1, 2	0, -1, -2	1	0
$S = \{\frac{1}{2^n}, n = 1, 2, \dots\}$	1, 2	0, -1, -2	$\frac{1}{2}$	0
$S = (-1, 1]$	1, 2	-1, -2, $-\pi$	1	-1
$S = [-1, 1]$	1, 2	-1, -2, $-e$	1	-1

2. (2 pts) For the sets below, find the following, if possible:

	two upper bounds	three lower bounds	sup S	inf S
$S = \{x \in \mathbb{R} : x^2 < 4x\}$	4, 5	0, -1	4	0
$S = \{x \in \mathbb{R} : x^2 \leq 1\}$	1, 2	-1, -2	1	-1

Show any work:

$$\begin{array}{l}
 x^2 < 4x \\
 \Rightarrow x > 0 \text{ and } x < 4 \\
 \text{or } x < 0 \text{ and } x > 4 \\
 \text{(impossible)} \\
 \hline
 \text{result: } x \in (0, 4)
 \end{array}
 \quad \left| \quad
 \begin{array}{l}
 |x| \leq 1 \\
 x \in [-1, 1]
 \end{array}$$

3. (2 pts) Show by mathematical induction that

$$S(n) : 1 + 3 + 5 + \dots + (2n - 1) = n^2$$

1^o Check: $1 \stackrel{?}{=} 1 \checkmark$

2^o $S(k) \stackrel{?}{\Rightarrow} S(k+1)$

$$S(k) : 1 + 3 + \dots + (2k - 1) = k^2$$

$$\begin{aligned}
 S(k+1) : 1 + 3 + \dots + (2k - 1) + (2(k+1) - 1) &= k^2 + 2k + 2 - 1 \\
 &= k^2 + 2k + 1 \\
 &= (k+1)^2 \checkmark
 \end{aligned}$$