MTH 351, Programming assignment 4, due 3/12

In this assignment you will explore numerical methods for solving linear systems and understand possible sources of instability. Please get familiar with MATLAB syntax for matrices and vectors. The following MATLAB routines may be helpful: GE pivot, Jacobi, GS, norm, cond.

1. Consider the following problem: find a least squares approximating polynomial \( q_n(x) \) of degree \( n \) for the function \( f(x) = \frac{1}{1+x} \) over the interval \([0,1]\) directly, that is, using the equation 4.113 in the textbook. The matrix that arises is called Hilbert matrix and it is ill-conditioned (that is why we do not usually solve LSQ problem using the method of undetermined coefficients for \( n > 2 \) but rather use orthogonal polynomials).

Write code in MATLAB that will deliver the solution. Note: you need to set up the matrix of the system \( A \) (use formula 4.113) and evaluate the right hand side vector \( b \). The right hand side vector can be computed using a quadrature rule applied to \( \int_0^1 x^i f(x) dx \). Use a) MATLAB routine quad which delivers very accurate values of \( b \), say \( b_{\text{quad}} \) and b) trapezoidal rule trapz which delivers much less accurate values of \( b \), especially if you use only a few points, say \( b_{\text{trap}} \).

To solve the system for the coefficients \( \alpha = (\alpha_0, \alpha_1, \alpha_n) \) of the polynomial \( q_n(x) = \alpha_0 + \alpha_1 x + \ldots + \alpha_n x^n \), use the MATLAB expression \( \alpha = A \ b \) and apply Gaussian elimination with and without pivoting.

Use \( n = 2, 5 \), and in the trapezoidal rule use \( n \) and \( 5n \) points.

Plot the function and the polynomials. Discuss the results with regard to the conditioning of the system. Hint: You may want to compare \( b_{\text{quad}} \) and \( b_{\text{trap}} \) depending on \( n \) and depending on the number of trapezoidal integration points.

2. Solve 6.6/6.