1. Show that trapezoidal method is of second order for non-autonomous case.

2. Consider the $\theta$-method defined as
   \[ U^{n+1} = U^n + h \left[ \theta f(U^n) + (1 - \theta) f(U^{n+1}) \right] \]
   for an autonomous ODE, where $0 \leq \theta \leq 1$ is a parameter.
   Verify that the local truncation error of the method is at least first-order.
   For what value of $\theta$ is the method second-order ?.

3. (MATLAB) Consider the IVP
   \[ f(u, t) = \lambda u + \sin(t), \quad y(0) = 1 \text{ for } 0 \leq t \leq 10. \]
   i) Implement FE and BE methods for this problem.
   ii) Plot the exact solution and the approximate solutions obtained with
        FE, BE with $h = 0.1$ and $h = 0.2$, when $\lambda = -5$. Discuss the behavior of
        the error from the plot.
   iii) Find the global error for each $h$ by taking $e_h := \max \{ |U^n - u(t_n)| \}$.
        Consider $h = 0.1, 0.01, 0.001$. Does the error behave as predicted by theory ?
        Compare how fast/slow the algorithm runs for various values of $h$.
        Extra: implement trapezoidal and midpoint methods and repeat iii).

4. Solve 5.9(b,c) or 5.13