MTH 452-552, Assignment 1

Students registered for 452 solve 1 (or all for extra credit). Students registered for 552 solve 2 problems.

1. Consider the *logistic* equation i.e. $u' = f(t, u) = ku(a - u)$ with initial condition $u(0) = u_0$, and with $k = 1, a = 4$.
   i) Discuss the well-posedness of this IVP (verify continuity and L-continuity, on what region is it satisfied, is the existence and uniqueness result global or local, formulate what we know from theory).
   ii) Find the analytical solution using any applicable method.
   iii) Plot (in MATLAB) or sketch by hand the direction fields and discuss the behavior of solutions for various values of $u_0 = -1, 0, 3, 5$. (Some of these solution are called equilibrium, attracting, and repelling solutions).
   Extra: what are the applications ? what is the meaning of constants k,a ? Consult available sources.

2. Consider the IVP $u'' + u = 0, u(0) = 1, u'(0) = 0$. Write it as a first order system. Solve it in both set-ups. What is the L-constant for this system ? (Consult Appendix A1-A3 for various matrix norms) . What are the eigenvalues of the matrix of this system ?
   Extra: what are the applications ? (Consult available sources.) Plot (by hand) solution to the original ODE and to the system. Relate both formulations.