

MTH 452-552/Winter 2013, Assignment 2, due Monday, 1/21  
All students solve all problems.

1. (MATLAB) Use the difference formulas  $D_-f$ ,  $D_0f$ ,  $\tilde{D}_+f$ , where the latter is the one-sided second-order accurate formula discussed in class, to approximate the derivative of  $f(x) = \cos(x)$  at  $x = .5$ . Use  $h$  ranging from  $1E-1$  down to  $1e-12$  (step by the factor of  $1/10$ ). Compare the approximation with the exact value. Discuss behavior of the error (confirm theoretical order of convergence and reveal instability which occurs for very small  $h$ ). (Use `loglog` plot).
2. (MATLAB) Consider the IVP  
 $f(u, t) = \lambda u + \sin(t)$ ,  $y(0) = 1$  for  $0 \leq t \leq 10$ .
  - i) Implement FE and BE methods for this problem.
  - ii) Plot the exact solution and the approximate solutions obtained with FE, BE with  $h = 0.1$  and  $h = 0.2$ , when  $\lambda = -5$ . Discuss the behavior of the error from the plot.
  - iii) Find the global error for each  $h$  by taking  $e_h := \max_n \{|U^n - u(t_n)|\}$ . Consider  $h = 0.1, 0.01, 0.001$ . Does the error behave as predicted by theory? Compare how fast/slow the algorithm runs for various values of  $h$ .
3. (EXTRA) Solve 1.1 from text.