MTH 453-553 S2013, Assignment 4

Students registered for 453 turn in 2abe, 3.
Students registered for 553 turn in 2, 3.
Do not turn in extra credit.
Use other problems as the basis for study for your Final exam.

Theoretical part.

1. Sketch the characteristics and the solution at $t = 1$ to $u_t - 2u_x = 0$, with the initial data $u_0(x) = max(0, 1 - |x|)$. Discuss what kind of solution this is (classical ?).

2. Derive the magnification factor $\rho(\nu)$ for the schemes: a) Lax-Friedrichs, b) FTCS, c) BTCS, d) BTBS, e) FTFS, using von-Neumann analysis. (Some calculations are outlined in the text already, in which case please work out the details on your own but do not turn them in).
   Plot $\rho(\nu)$ in the complex plane and discuss the region of stability. You should do this for various values of ν.
   Derive the conditions for stability. Take into account the sign of a; propose the choice of k depending on h that make the scheme stable.

3. Derive the LTE for the Lax-Friedrichs scheme. Knowing from pbm 2 (and from textbook) that its stability condition requires that $k = O(h)$ (verify the exact form of the condition), show that the scheme is actually first oder accurate in h.

4. Derive LTE for all the schemes discussed in class and confirm the order of accuracy that we claimed in class.

5. Derive/review stability conditions for all the schemes discussed in class using the Method of Lines and von-Neumann analysis.

Computational part is due Friday June 7. You can use the code `advection1d.m` and modify as needed.

6. Use the upwind (FB) scheme to solve $u_t + u_x = 0$. Use $\lambda = .95$, and test its convergence in L^2 grid norm with the following initial data: a) $u_0(x) = sin(x)$, b) $u_0(x) = max(0, 1-|x|)$, c) $u_0(x) = H(x-1)-H(x-2)$, where $H(x)$ is the Heaviside function.

   Repeat with L^1 grid norm and L^∞ norm. Some of the answers may surprise you, but please consider the smoothness of the solutions and how it relates to the local truncation error.

   Extra: use other λ. [In particular, try $\lambda = 1$.]

7. Implement the downwind scheme for the PDE $u_t - 2u_x = 0$ with $u_0(x) = sin(x)$. Show its first order convergence in L^2 grid norm when $\lambda = .95$.

   Try also different $\lambda = .4$, $\lambda = .6$, $\lambda = 2.1$. Explain whether the convergence still holds, and why/or why not. Note that $\lambda = \frac{k}{h}$ and the stability condition for schemes for $u_t + au_x = 0$ involves $\nu = a\lambda$.