MTH 453-553/Peszynska **Final, due Friday 6/7 in class**

Name ______________________________

Student ID # ______________________________

Instructions: Individual work only. WRITE NEATLY ! Sloppy exams will not be graded. Use your own paper for solutions.

453 students solve 1b, 2a, and 3. **553** students solve 1a, 2ab, and 3.

1.(453 solve part b), 553 solve part a)), 10 points

[**553**] a) Propose a difference scheme for advection-diffusion equation

$$u_t + au_x - Du_{xx} = 0 \tag{1}$$

which approximates the diffusion explicitly in time, to second order in space, and advection to second order in space, implicitly in time.

Determine its LTE. Conjecture what would be stability requirements on k.

[**453**] b) Propose a second order unconditionally stable scheme for the advection-reaction equation

$$u_t + au_x + Ru = 0 \tag{2}$$

Show details of accuracy and stability analysis.

2.(453 and 553), 10 points

(453 and 553) a) Plot the stability region in the complex plane for the following hypothetical "FinalExam-scheme" in which the growth (amplification) factor is

$$\rho = -\frac{1}{2} + i\nu\cos(h\xi). \tag{3}$$

Determine the stability condition in terms of $\nu = a\frac{k}{h}$.

(553) b) Consider the explicit scheme $u_j^{n+1} = \alpha u_{j-1}^n + \beta u_{j+1}^n$ for the advection equation. Show that the scheme is stable if and only if $|\alpha| + |\beta| \leq 1$.

3.(453 and 553), 10 points

You are given the following incomplete convergence results from a code that implements some hypothetical schemes A-C for the "two-way" wave equation $u_{tt} = u_{xx}$ on $x \in (0,1), t > 0$.

Ideally, each scheme should be at least $O(h^2 + k^2)$. As long as the scheme is stable, convergence order should be optimal.

Find the possible values for the missing information in the following tables. **Explain**.

	h	k	error	order
Scheme (A) unconditionally stable	0.1	1	0.23	$O(h^2 + k^2)$
	0.01		0.00225	-

	h	k	error	
Scheme (B) (stable if $k \leq h$)	0.1		5.67	$O(h^2 + k^2)$
	0.01	0.0001	6.1	-

	h	k	error	order
Scheme (C) (unconditionally stable)	0.1	0.1	0.3	$O(h^4 + k^2)$
	0.01			-