equation at first, but then try the following to verify your observations rigorously:

1. First fix $a=1$. Use the graph of $f_{1,b}$ to construct the bifurcation diagram for this family of differential equations depending on b.
2. Repeat the previous question for $a=0$ and then for $a=-1$.
3. What does the bifurcation diagram look like for other values of a?
4. Now fix b and use the graph to construct the bifurcation diagram for this family, which this time depends on a.
5. In the ab–plane, sketch the regions where the corresponding differential equation has different numbers of equilibrium points, including a sketch of the boundary between these regions.
6. Describe, using phase lines and the graph of $f_{a,b}(x)$, the bifurcations that occur as the parameters pass through this boundary.
7. Describe in detail the bifurcations that occur at $a=b=0$ as a and/or b vary.
8. Consider the differential equation $x' = x - x^3 - b\sin(2\pi t)$, where $|b|$ is small. What can you say about solutions of this equation? Are there any periodic solutions?
9. Experimentally, what happens as $|b|$ increases? Do you observe any bifurcations? Explain what you observe.

EXERCISES

1. Find the general solution of the differential equation $x' = ax + 3$ where a is a parameter. What are the equilibrium points for this equation? For which values of a are the equilibria sinks? For which are they sources?
2. For each of the following differential equations, find all equilibrium solutions and determine whether they are sinks, sources, or neither. Also sketch the phase line.
   (a) $x' = x^3 - 3x$
   (b) $x' = x^4 - x^2$
   (c) $x' = \cos x$
   (d) $x' = \sin^2 x$
   (e) $x' = |1 - x^2|$
3. Each of the following families of differential equations depends on a parameter a. Sketch the corresponding bifurcation diagrams.
   (a) $x' = x^2 - ax$

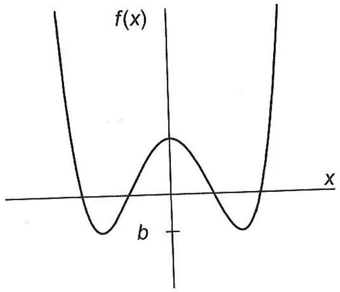

Figure 1.12 Graph of the function f.

4. Consider the function $f(x)$ with a graph that is displayed in Figure 1.12.
   (a) Sketch the phase line corresponding to the differential equation $x' = f(x)$.
   (b) Let $g_a(x) = f(x) + a$. Sketch the bifurcation diagram corresponding to the family of differential equations $x' = g_a(x)$.
   (c) Describe the different bifurcations that occur in this family.
5. Consider the family of differential equations

$$x' = ax + \sin x,$$

where a is a parameter.
   (a) Sketch the phase line when $a = 0$.
   (b) Use the graphs of ax and $\sin x$ to determine the qualitative behavior of all of the bifurcations that occur as a increases from -1 to 1.
   (c) Sketch the bifurcation diagram for this family of differential equations.
6. Find the general solution of the logistic differential equation with constant harvesting,

$$x' = x(1-x) - h,$$

for all values of the parameter $h > 0$.
7. Consider the nonautonomous differential equation

$$x' = \begin{cases} x - 4 & \text{if } t < 5, \\ 2 - x & \text{if } t \geq 5. \end{cases}$$

   (a) Find a solution of this equation satisfying $x(0) = 4$. Describe the