1. Discuss existence and uniqueness of solutions to the following BVP:

\[ u'' = x, \ x \in (0, 1), \]

with homogeneous a) Dirichlet and b) Neumann boundary conditions. Does the answer change when you consider the problem posed on \((-1, 1)\) instead on \((0, 1)\)?

2. Let \(u\) solve \(u'' + ku = 0\), where \(k > 0\). Consider an IVP for this equation with \(u(0) = 0, u'(0) = 1\) given. Consider a BVP for this equation with \(u(0) = 0, u(1) = 0\). Discuss the well-posedness for both cases.

3. Find and sketch the regions in the \(xy\) plane where the equation

\[ (1 + x)u_{xx} + 2xyu_{xy} - y^2u_{yy} = 0 \]

is elliptic, hyperbolic, or parabolic.

4. Let \(k\) be an arbitrary real constant. Depending on \(k\), determine the type and transform the equation \(u_{xx} + 2u_{xy} + ku_{yy} = 0\) where \(k\) to a canonical form, by changing variable. Propose the general solution whenever possible.