

MTH 621/Peszynska/Fall 2011, Assignment 1

Please show all your work. Use proper mathematical notation.

1. Let  $v$  be a constant.
  - 1) Find general solution to  $u_t + vu_x = 0$ .
  - 2) Sketch characteristics: consider  $v = 0, v = -1, v = 1/2$ .
  - 3) Sketch the solution in  $(x, u)$  plane for the initial condition  $u(x, 0) = \frac{1}{1+x^2}$  for  $t = 1, 2, 5$ .
2. Find and sketch characteristics for  $(x^2 + 1)u_x + u_y = 0$ . Suggest an auxiliary condition so that a solution can be found in some region  $D \subseteq \mathbb{R}^2$ . Also, suggest a condition for which a solution cannot be found.
3. Solve the equation  $u_x + u_y = 1$  with the condition a)  $u(x, 0) = 5$  and b)  $u(0, y) = \max(0, 1 - y^2)$ , if possible. If not possible, explain why. What behavior of solutions do you expect? (Answer before and after you found the solution.)