MTH 621/Peszynska/Fall 2011, Assignment 3

Please show enough of your work to justify the answer but be concise.
Use proper mathematical notation.

1. Verify directly that the solution of the non-homogeneous wave equation in $\mathbb{R}$ given in class works (there is an additional multiplicative constant involved). Discuss the smoothness of the data necessary for that solution to make sense.

2. Sketch the formal solution to the homogeneous wave equation with "box" initial data, $c = 1$, at $t = 0, t = 1/2, t = 1, t = 2$. How does $\int_{-\infty}^{\infty} u(x,t)dx$ change with t ?

3. Consider the formal solution to the homogeneous wave equation with "sine" initial data with $c = 1$ and show that the total energy

$$E(t) = \frac{1}{2} \int_{\mathbb{R}} \left((u_t)^2 + (u_x)^2 \right) dx \tag{0.1}$$

does not change. (For the energy to be finite, choose the support of the "sine" initial data to be, e.g., in $(0, \pi)$.
Note: the first part of the integral gives the kinetic energy, and the second the potential energy. Thus, the wave equation expresses conservation of momentum.
Challenge: show that this is true for any compactly supported initial data so that the solution and its derivatives vanish at $+/-\infty$. (Hint: consider $\frac{dE}{dt}$ and integrate by parts several times).

4. Solve 4-2.11. You can replace (e-f) by any analytical or computational method of your choice; include the references if you use any sources.