MTH 654/659 Fall 2011. Assignment 2 (lab 1)

Instructions: Please follow the instructions below and try to get through as much as possible during the lab. Ask me questions if you get stuck or need help. If all goes right in problems 1,2, you should have no trouble with the third. Please write a (concise) lab report on your solution to the third problem. [The assignments are designed in MATLAB but if you prefer a different language/environment, go ahead.]

Problems

1. Let $h > 0$ be a parameter. Assume you have a method that converges with order α and rate C that is, whose error

$$e(h) \approx Ch^{\alpha} \tag{0.1}$$

   Practice how to assess that convergence with each of the methods below (generally one is sufficient, but you should compare how they work).
   - Use `log-log` (and `grid on`) plot of e versus h
   - Fit e to h (well, their logs)
   - Solve for α and C (take a log of (0.1) and use the data in the table)
   - Use a polynomial fit of all data if α is expected to be an integer

   Use the data below

   i)

h	0.1	0.05	0.025	0.01
e	0.0498	0.012441	0.003158	0.000506

   ii)

h	0.1	0.05	0.025	0.01
e	0.061265	0.02234	0.007862	0.0020293

2. Get familiar with numerical integration. Verify (quadratic) order of convergence of trapezoidal method: use `trapz` or write your own code. Compare the results with those using the more accurate (and adaptive) `quad` function. For example, consider the exact and approximate values of $\int_0^{\pi} sin(x)dx$. What about $\int_0^1 \sqrt{x}dx$?
3. Demonstrate the accuracy of piecewise linear interpolation with `interp1`. In class, I stated the following results concerning the interpolation error

$$\max_{x \in (0,1)} |u(x) - I_h u(x)| \leq C_{max} h^2$$
$$\| u - I_h u \|_{L^2(0,1)} \leq C_{L^2} h^2$$
$$\| u - I_h u \|_{H^1(0,1)} \leq C_{H^1} h$$

   where C_{max} depends on $\| u'' \|_{C^0}$, and C_{L^2}, C_{H^1} depend on $\| \partial^2 u \|_{L^2}$. (Proof of these results can be found in [Atkinson, Han'05, Chap.3.2]). For example, consider $sin(x)$ on $(0, \pi)$. What about $\sqrt{x}$ on $(0, 1)$?