(I1) Give an example of a metric space that is bounded but not totally bounded. Verify both assertions.

(I2) Let \( \Omega = \{0, 1\}^\mathbb{N} \). Fix \( w \in (0, 1)^\mathbb{N} \) with \( \sum_{k=1}^{\infty} w_k = 1 \) and define \( d_w : \Omega \times \Omega \to \mathbb{R} \) as

\[
d_w(x, y) = \sum_{k=1}^{\infty} w_k |x_k - y_k|.
\]

Note: you can think of \( w \) as a vector of weights that generates \( d_w \).

(a) Verify that \( (\Omega, d_w) \) is a metric space.

(b) Find \( \text{diam} \, \Omega \) in \( d_w \).

(c) For \( k \geq 0 \), let \( \Omega_k = \{x \in \Omega : x_j = 0 \text{ for } j > k\} \). Find card \( \Omega_k \).

(d) For \( k \geq 0 \), let \( \epsilon_k = \sum_{j>k} w_j \). Show that for any \( \epsilon > \epsilon_k \), \( \Omega_k \) is an \( \epsilon \)-net for \( (\Omega, d_w) \).

(e) Prove or disprove: \((\Omega, d_w)\) is totally bounded.

(f) Prove or disprove: \((\Omega, d_w)\) is complete.

(g) Recall that two metrics are equivalent if they generate the same convergent sequences. (See page 48 of your text.) Let \( d_w \) and \( d_v \) be metrics on \( \Omega \) generated respectively by the weight vectors \( w \) and \( v \) where \( w \neq v \) with both \( w, v \in (0, 1)^\mathbb{N} \). Show that \( d_w \) and \( d_v \) are equivalent.

(h*) In (g) we required that the weight vectors \( w, v \) be in \( (0, 1)^\mathbb{N} \) with \( \sum_{k=1}^{\infty} w_k = \sum_{k=1}^{\infty} v_k = 1 \). If we instead relax this requirement to \( v, w \in [0, 1]^\mathbb{N} \) with \( \sum_{k=1}^{\infty} w_k = \sum_{k=1}^{\infty} v_k = 1 \), will \( d_w \) and \( d_v \) still be equivalent for any such \( v \neq w \)?