

MTH 665: Probability Theory 3 credits Section 001, Spring 2018

Catalog Description: General theory of probability measures and random variables, including weak convergence, characteristic functions, central limit theory, conditional expectations, martingales.

Prerequisites: MTH 664 or an equivalent course in measure theoretic probability.

Meets: Three 50 minute lectures weekly.

Instructor: Professor Ossiander, Kidder 298B, ossiand@math.oregonstate.edu

Office Hours: Monday and Wednesday 1:30-2:30pm and Tuesday 12:30-1:30pm. Also available by appointment.

Course Content: This class is the second in a two term sequence in probability theory. It is intended for graduate students who have a foundation in measure theoretic probability. The goal of the course is to continue the study of probability using the backgrounds and tools developed in MTH 664. Topics are as follows:

- Convergence in distribution of random variables
- Characteristic functions; existence and uniqueness
- Central limit theorems, including the Lindeberg-Levy
- Stable and infinitely divisible distributions
- Conditional expectation from a measure theoretic point of view
- Martingales and martingale convergence
- Existence and characterization of stochastic processes
- Brownian motion

These topics correspond to the following sections of Chapters 5 through 7 of the Billingsley text.

- Chapter 5
 - Section 25, Weak Convergence
 - Section 26, Characteristic Functions
 - Section 27, The Central Limit Theorem
 - Section 28, Infinitely Divisible Distributions
 - Section 29, Limit Theorems in \mathbf{R}^k (covered briefly)
- Chapter 6
 - Section 32, The Randon-Nikodym Theorem (covered briefly)
 - Section 33, Conditional Probability
 - Section 34, Conditional Expectation
 - Section 35, Martingales

- Chapter 7
 - Section 36, Kolmogorov’s Existence Theorem
 - Section 37, Brownian Motion

Learning Resources: The required text for the course is *Probability and Measure*, by Patrick Billingsley, Anniversary Edition, published by Wiley, 2012. This is a very nice text book that covers the foundational concepts and results of probability theory. It will be followed selectively. (There are some differences between the ‘Anniversary Edition’ and the earlier first, second, and third editions.) Students may find Jeffrey S. Rosenthal’s *A First Look at Rigorous Probability Theory* helpful as an auxiliary text, but it is not required.

Course Plan: This course is lecture based. Homework will be assigned bi-weekly. Students are expected to write up homework solutions independently. Late homework is strongly discouraged and may be penalized. The last set of homework problems may be comprehensive. No examinations will be given.

Learning Outcomes: Upon completing MTH 665 a successful student is expected to be able to do the following.

1. The ability to use and build on the skills and knowledge gained in MTH 664 to further study weak convergence of random variables.
2. Compute characteristic functions, evaluate the limits of sequences of characteristic functions, and understand the ramifications in terms of weak convergence of random variables.
3. Apply the Lindeberg-Levy Central Limit Theorem to sums of independent random variables.
4. Calculate conditional expectations in the measure-theoretic setting.
5. Recognize martingales and semi-martingales and apply martingale convergence theory.
6. Derive properties of Brownian motion on the real line.

Evaluation of Student Learning: (Approximate percentages given.)

- Homework problems 100 %

Students with Disabilities: Accommodations are collaborative efforts between students, faculty and Disability Access Services (DAS). Students with accommodations approved through DAS are responsible for contacting the faculty member in charge of the course prior to or during the first week of the term to discuss accommodations. Students who believe they are eligible for accommodations but who have not yet obtained approval through DAS should contact DAS immediately at 541-737-4098.

Student Conduct: All students are expected to obey OSU’s student conduct regulations. Here is the link to OSU’s Statement of Expectations for Student Conduct:
<http://studentlife.oregonstate.edu/studentconduct/offenses-0>