The widespread availability of personal computers has brought powerful computational and graphical capability within the reach of individual students. You should consider, in the light of your own circumstances, how best to take advantage of the available computing resources. You will surely find it enlightening to do so.

Another aspect of computer use that is very relevant to the study of differential equations is the availability of extremely powerful and general software packages that can perform a wide variety of mathematical operations. Among these are Maple, Mathematica, and MATLAB, each of which can be used on various kinds of personal computers or workstations. All three of these packages can execute extensive numerical computations and have versatile graphical facilities. Maple and Mathematica also have very extensive analytical capabilities. For example, they can perform the analytical steps involved in solving many differential equations, often in response to a single command. Anyone who expects to deal with differential equations in more than a superficial way should become familiar with at least one of these products and explore the ways in which it can be used.

For you, the student, these computing resources have an effect on how you should study differential equations. To become confident in using differential equations, it is essential to understand how the solution methods work, and this understanding is achieved, in part, by working out a sufficient number of examples in detail. However, eventually you should plan to delegate as many as possible of the routine (often repetitive) details to a computer, while you focus on the proper formulation of the problem and on the interpretation of the solution. Our viewpoint is that you should always try to use the best methods and tools available for each task. In particular, you should strive to combine numerical, graphical, and analytical methods so as to attain maximum understanding of the behavior of the solution and of the underlying process that the problem models. You should also remember that some tasks can best be done with pencil and paper, while others require a calculator or computer. Good judgment is often needed in selecting a judicious combination.

**PROBLEMS**

In each of Problems 1 through 6 determine the order of the given differential equation; also state whether the equation is linear or nonlinear.

1. \( r \frac{d^2 y}{dt^2} + t \frac{dy}{dt} + 2y = \sin t \)
2. \( (1 + y^2) \frac{d^2 y}{dt^2} + t \frac{dy}{dt} + y = e^t \)
3. \( \frac{d^3 y}{dt^3} + \frac{d^2 y}{dt^2} + \frac{dy}{dt} + y = 1 \)
4. \( \frac{dy}{dt} + ty^2 = 0 \)
5. \( \frac{d^2 y}{dt^2} + \sin(t + y) = \sin t \)
6. \( \frac{d^2 y}{dt^2} + t \frac{dy}{dt} + (\cos^2 t)y = t^3 \)

In each of Problems 7 through 14 verify that each given function is a solution of the differential equation.

7. \( y'' - y = 0; \quad y_1(t) = e^t, \quad y_2(t) = \cosh t \)
8. \( y'' + 2y' - 3y = 0; \quad y_1(t) = e^{-t}, \quad y_2(t) = e^t \)

In each of Problems 15 through 18 determine whether the equation has solutions of the form \( y = e^{rt} \).

15. \( y'' + 2y = 0 \)
16. \( y'' + y = 0 \)
17. \( y'' - 5y = 0 \)
18. \( y'' + y = 0 \)

In each of Problems 19 and 20 determine whether the equation has solutions of the form \( y = \cos xt \).

19. \( y'' + 4y = 0 \)
20. \( y'' + y = 0 \)

In each of Problems 21 through 24 determine whether the equation has solutions of the form \( y = e^{ax} \).

21. \( u_{xx} + u_{yy} + u_{tt} = 0 \)
22. \( u_{xx} + 2u_{xy} + u_{yy} = 0 \)
23. \( u_{xx} + u_{yy} = 0 \)
24. \( u_{xx} + 2u_{y} + u_{tt} = 0 \)

In each of Problems 25 through 28 verify that each given function is a solution of the partial differential equation.

25. \( u_{xx} + u_{yy} = 0; \quad u(t, x) = \cos x \sin \theta \)
26. \( \alpha^2 u_{xx} = u_t; \quad u(t, x) = e^{\alpha^2 t} \sin \alpha x \)
27. \( \alpha^2 u_{xx} = u_t; \quad u(t, x) = \sin \alpha x \sin \alpha t \)
28. \( \alpha^2 u_{xx} = u_t; \quad u(t, x) = (\alpha/\sqrt{\pi}) e^{-x^2/\alpha^2} \)
29. Follow the steps indicated here to verify the solution in the text. Assume that the rod is in an environment where there is no friction or drag and that there is no initial tension.

(a) Assume that the mass is in an environment where there is no friction or drag and that there is no initial tension.

(b) Apply Newton's law of motion to the mass. Then the tensile force is given by the equation \( F = \frac{d^2 \theta}{dt^2} \), where \( L_d \theta / dt^2 \) is the length of the rod.

(c) Simplify the result from part (a).

30. Another way to derive the pendulum equation of energy.

(a) Show that the kinetic energy is given by \( T = \frac{1}{2} L_d^2 \theta^2 \).

(b) Show that the potential energy is given by \( U = \frac{1}{2} L_d \theta^2 \).

(c) Simplify the result from part (b).
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9. \[ ty' - y = r^2; \quad y = 3t + r^2 \]

10. \[ y''' + 4y'' + 3y' = 0; \quad y_1(t) = r \quad y_2(t) = e^{r} + r^2 \]

11. \[ 2t^2y'' + 3ty' - y = 0; \quad t > 0; \quad y_1(t) = 1/2, \quad y_2(t) = t^{-1} \]

12. \[ t^2y'' + 5ty' + 4y = 0; \quad t > 0; \quad y_1(t) = r^2, \quad y_2(t) = r^{-2} \ln t \]

13. \[ y'' + y = \sec t, \quad 0 < t < \pi/2; \quad y = (\cos t) \ln \cos t + r \sin t \]

14. \[ y' - 2ty = 1; \quad y = e^x \int_0^x e^{-s} ds + e^x \]

In each of Problems 15 through 18 determine the values of \( r \) for which the given differential equation has solutions of the form \( y = e^{rt} \).

15. \( y' + 2y = 0 \)

16. \( y'' - y = 0 \)

17. \( y'' + y' - 6y = 0 \)

18. \( y'' - 3y' + 2y = 0 \)

In each of Problems 19 and 20 determine the values of \( r \) for which the given differential equation has solutions of the form \( y = e^{rt} \) for \( t > 0 \).

19. \( t^2y'' + 4ty' + 2y = 0 \)

20. \( t^2y'' - 4ty' + 4y = 0 \)

In each of Problems 21 through 24 determine the order of the given partial differential equation; also state whether the equation is linear or nonlinear. Partial derivatives are denoted by subscripts.

21. \( u_{xx} + u_{yy} + u_{zz} = 0 \)

22. \( u_{xx} + u_{xy} + u_{yx} + u_{yy} + u = 0 \)

23. \( u_{xxx} + 2u_{xxy} + u_{yyy} = 0 \)

24. \( u_t + u_{tt} = 1 + u_x \)

In each of Problems 25 through 28 verify that each given function is a solution of the given partial differential equation.

25. \( u_{xx} + u_{yy} = 0; \quad u_1(x, y) = \cos x \cosh y, \quad u_2(x, y) = \ln (x^2 + y^2) \)

26. \( \alpha^2 u_{xx} = u_t; \quad u_1(x, t) = e^{-\alpha^2} \sin \alpha x, \quad u_2(x, t) = e^{-\alpha^2} \sin \alpha t, \quad \alpha \text{ a real constant} \)

27. \( \alpha^2 u_{xx} = u_t; \quad u_1(x, t) = \sin \lambda x \sin \lambda t, \quad u_2(x, t) = \sin (x - at), \quad \lambda \text{ a real constant} \)

28. \( \alpha^2 u_{xx} = u_t; \quad u = (\pi/2)^{1/2} e^{-\alpha^2} \sin \alpha t, \quad t > 0 \)

29. Follow the steps indicated here to derive the equation of motion of a pendulum, Eq. (12) in the text. Assume that the rod is rigid and weightless, that the mass is a point mass, and that there is no friction or drag anywhere in the system.

   (a) Assume that the mass is in an arbitrary displaced position, indicated by the angle \( \theta \). Draw a free-body diagram showing the forces acting on the mass.

   (b) Apply Newton's law of motion in the direction tangential to the circular arc on which the mass moves. Then the tensile force in the rod does not enter the equation. Observe that you need to find the component of the gravitational force in the tangential direction. Observe also that the linear acceleration, as opposed to the angular acceleration, is \( L \dot{\theta}/d\theta \), where \( L \) is the length of the rod.

   (c) Simplify the result from part (b) to obtain Eq. (12) in the text.

30. Another way to derive the pendulum equation (12) is based on the principle of conservation of energy.

   (a) Show that the kinetic energy \( T \) of the pendulum in motion is

   \[ T = \frac{1}{2} mL^2 \left( \frac{d\theta}{dt} \right)^2. \]

   (b) Show that the potential energy \( V \) of the pendulum, relative to its rest position, is

   \[ V = mgL(1 - \cos \theta). \]