Modeling the Catchment Via Mixtures: an Uncertainty Framework for Dynamic Hydrologic Systems

Lucy Marshall

Assistant Professor of Watershed Analysis

Department of Land Resources and Environmental Sciences

Montana State University

Email: lmarshall@montana.edu

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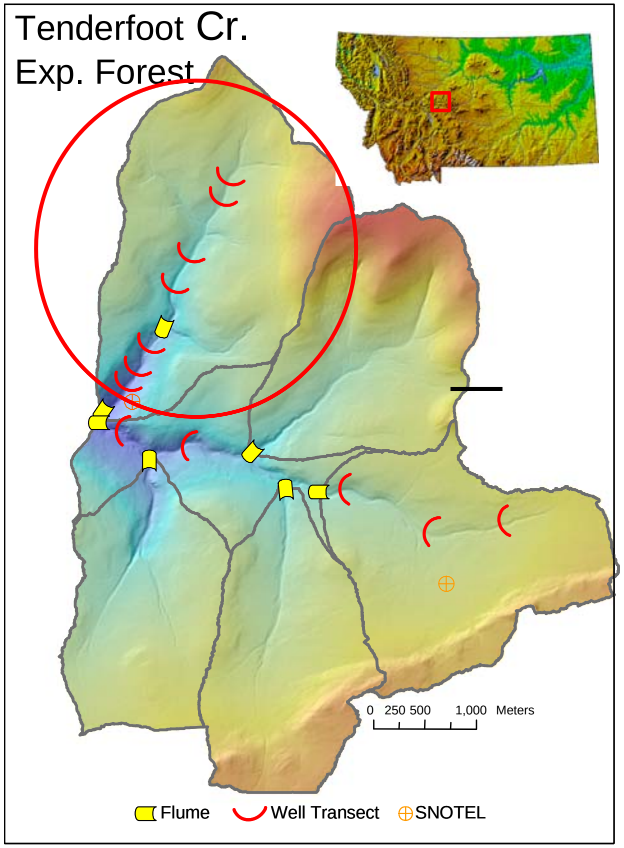
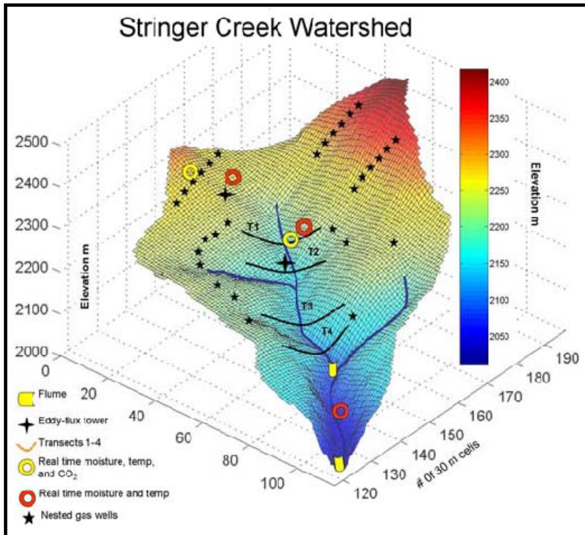
Ashish Sharma- University of New South Wales
David Nott- National University of Singapore

Conceptualizing first order watershed processes

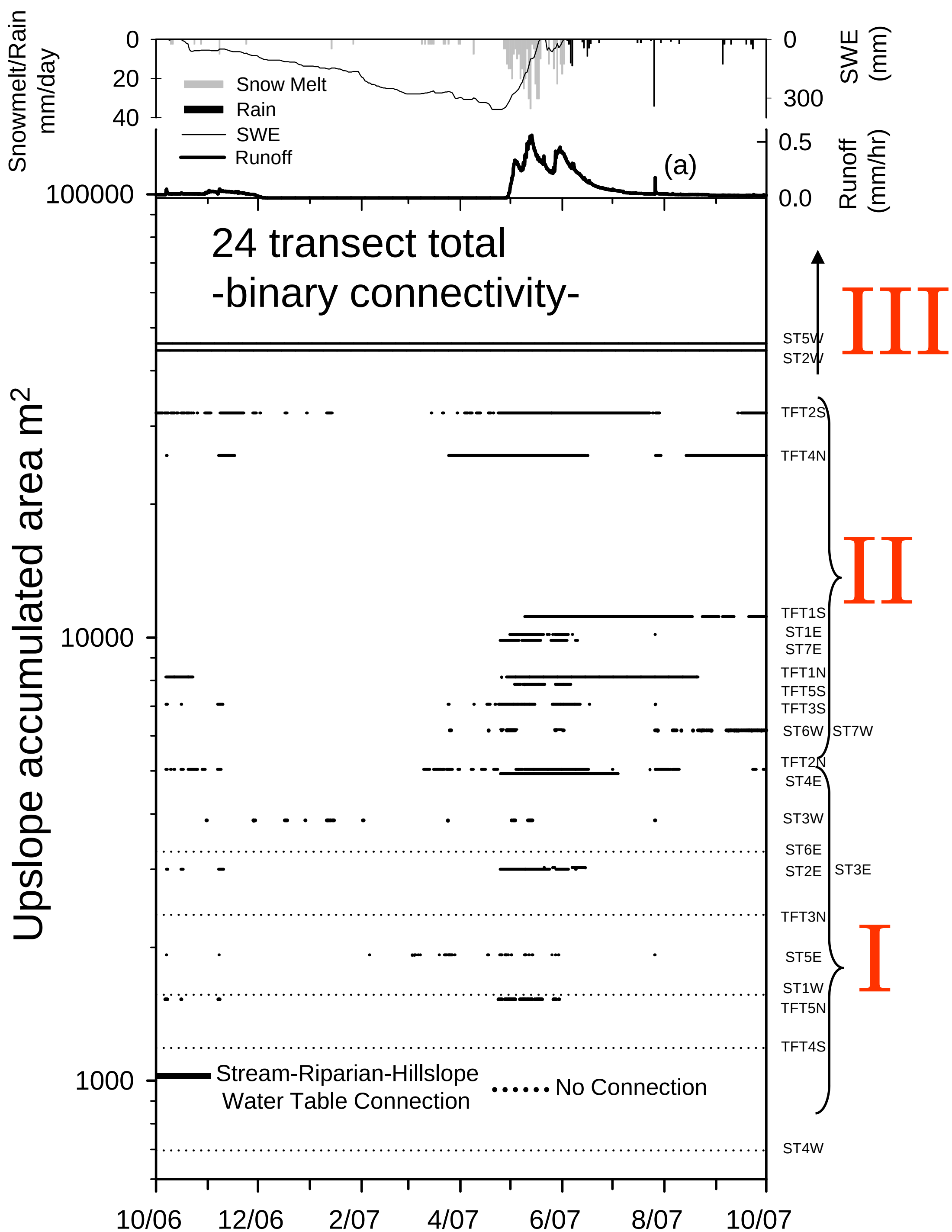


The Tenderfoot Creek Experimental Forest

- 7 nested watersheds
- Lodgepole pine vegetation
- Melt driven runoff
- Freezing temperatures can occur in every month



- 555 ha
- Full range of slope and topographic convergence, divergence
- Elevation ranges from 1840m to 2420



Winter

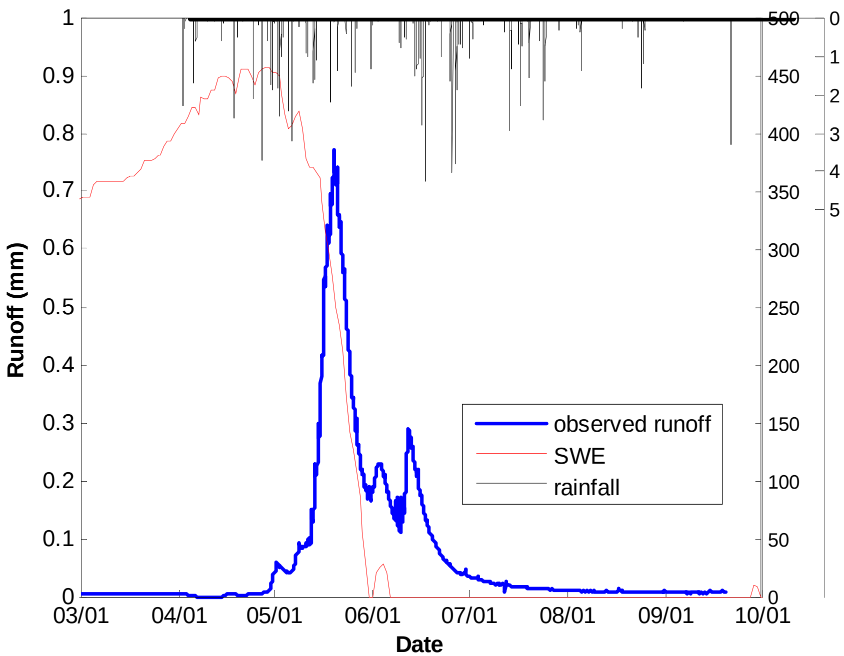
Spring

Summer

Kelsey Jencso

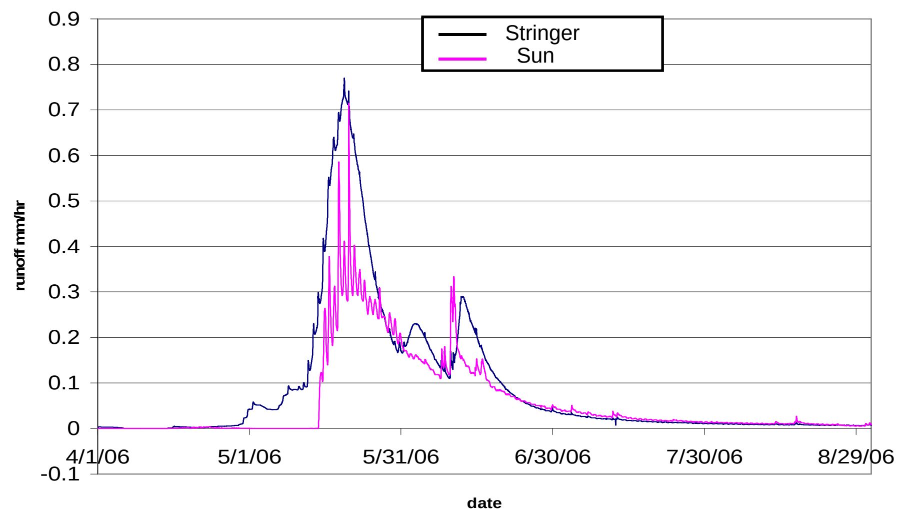
Conceptualizing first order watershed processes

Unknown Process/Model Implementation



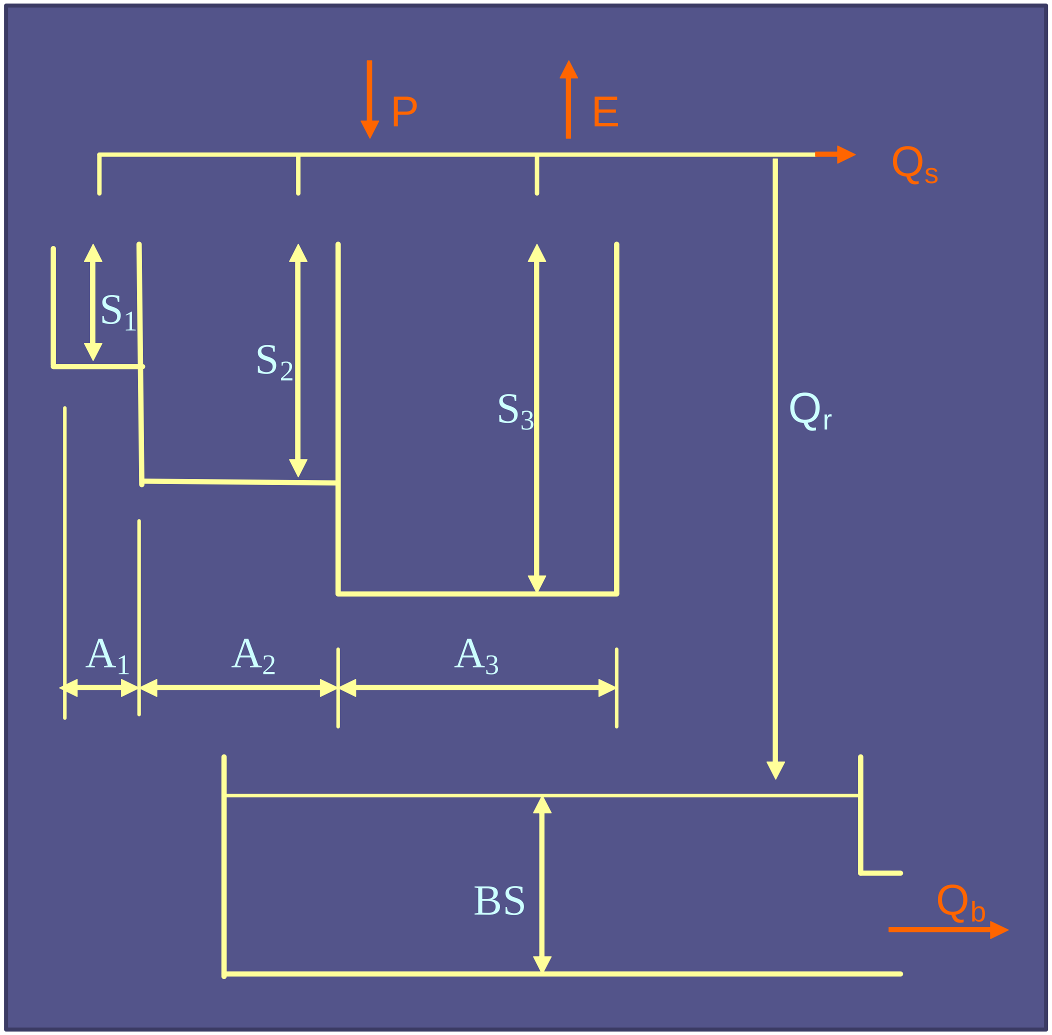
Snow melt → -Temp/energy dependent? -Elevation effects? -Rain on snow?

Soil Moisture Accounting/sub surface flow → -Thresholds? -Seasonal?

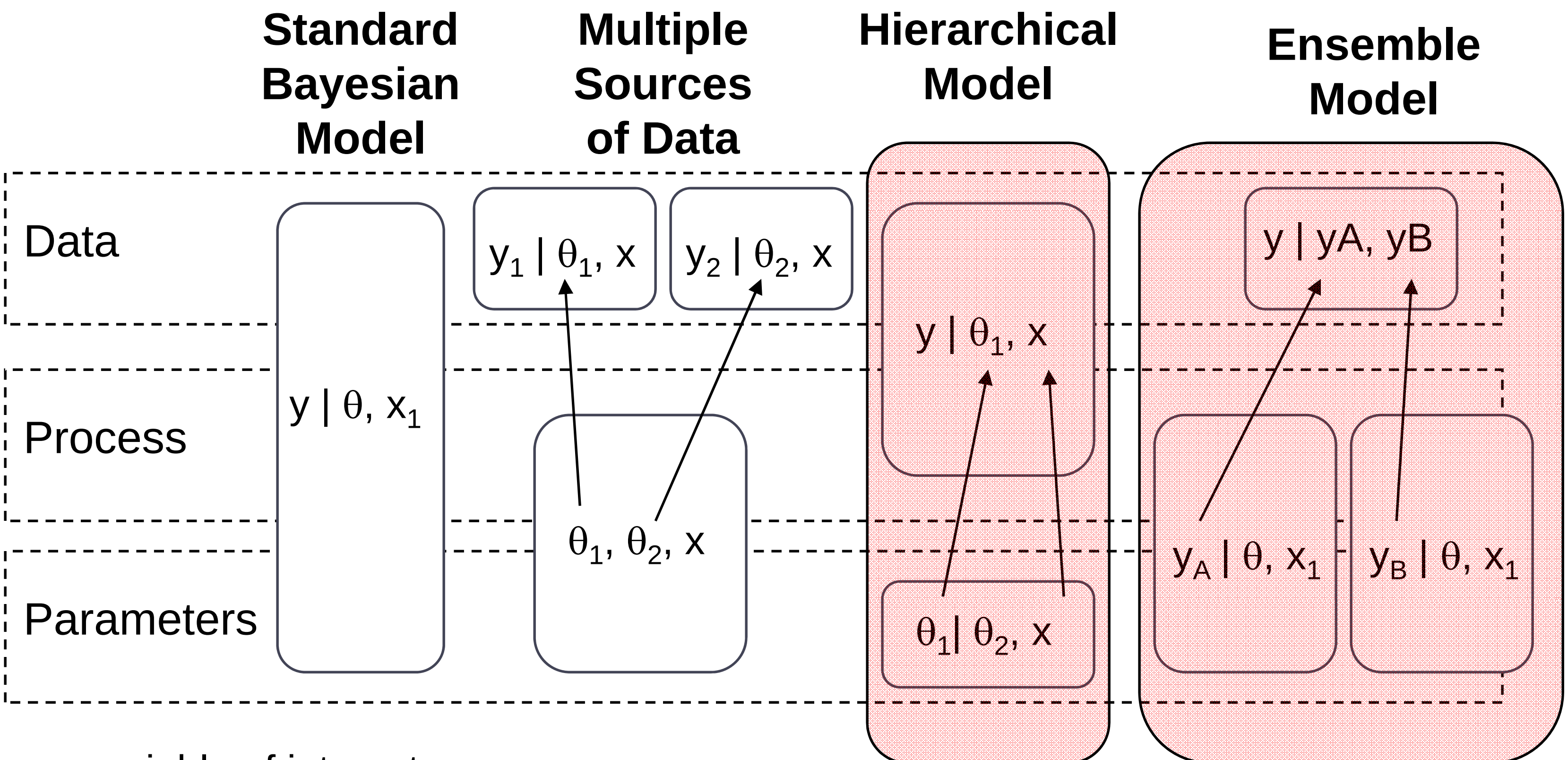


Conceptual rainfall-runoff modeling in hydrology

- Watershed is represented as a variable series of storages.
- Model uses rainfall, evapotranspiration etc. time series as inputs to simulate runoff
- Conceptual distributed models: discretize catchment into individual units, or use hydrologic response units



An uncertainty framework – ways of incorporating hydrologic complexity

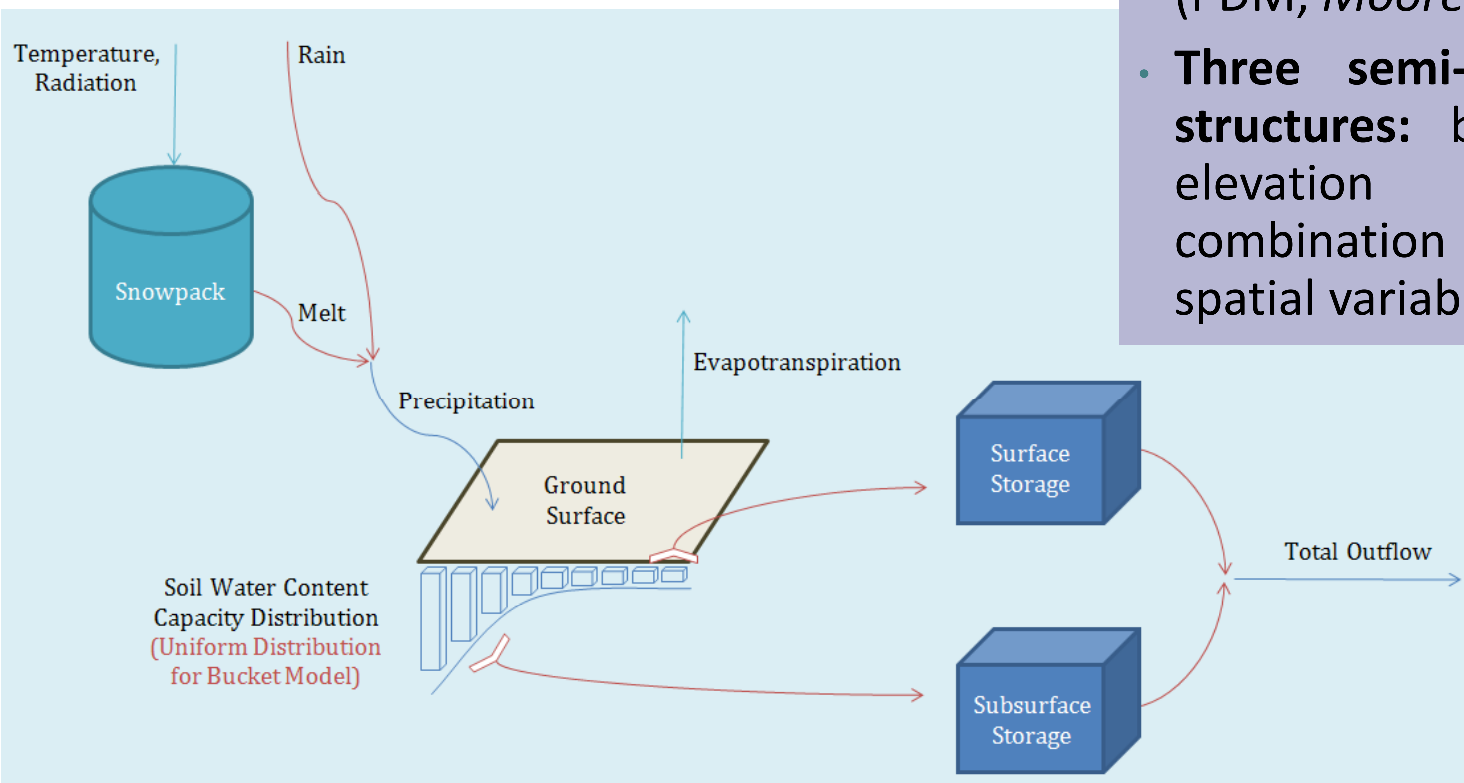


y ~ variable of interest
x ~ input data, climatological variables
θ ~ parameters

Adapted after Clark, Ecology Letters, 2005.

Base conceptual models

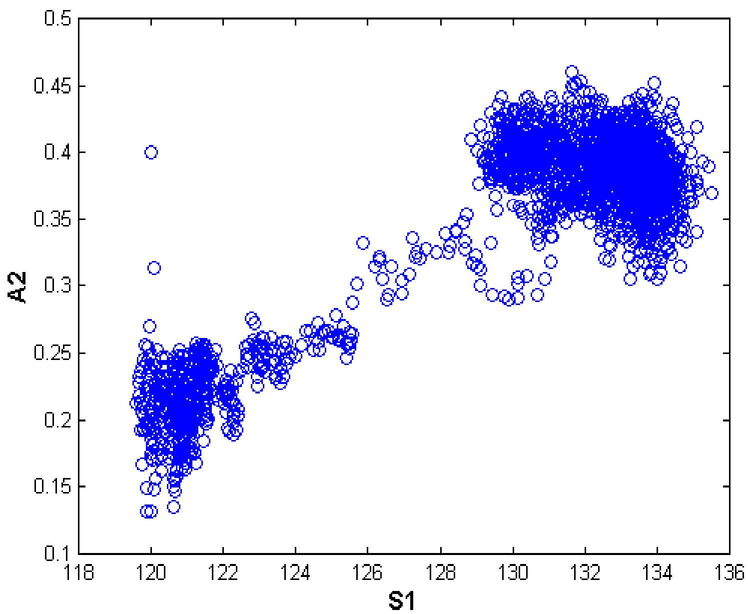
- **Two base structures**: a simple bucket model and the probability distributed model (PDM, *Moore*, 1985).
- **Three semi-distribution sub-structures:** based on aspect, elevation and their combination to account for spatial variability in inputs.

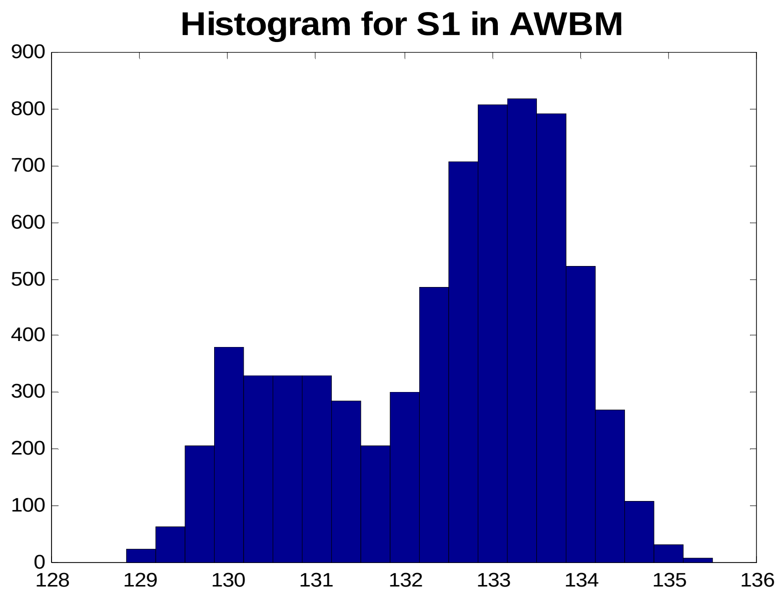


- **Three snowmelt accounting routines:** temperature index, radiation index and the combination.

Difficulties in characterizing hydrologic model uncertainty

- Hydrological models: often have highly correlated and interdependent parameters

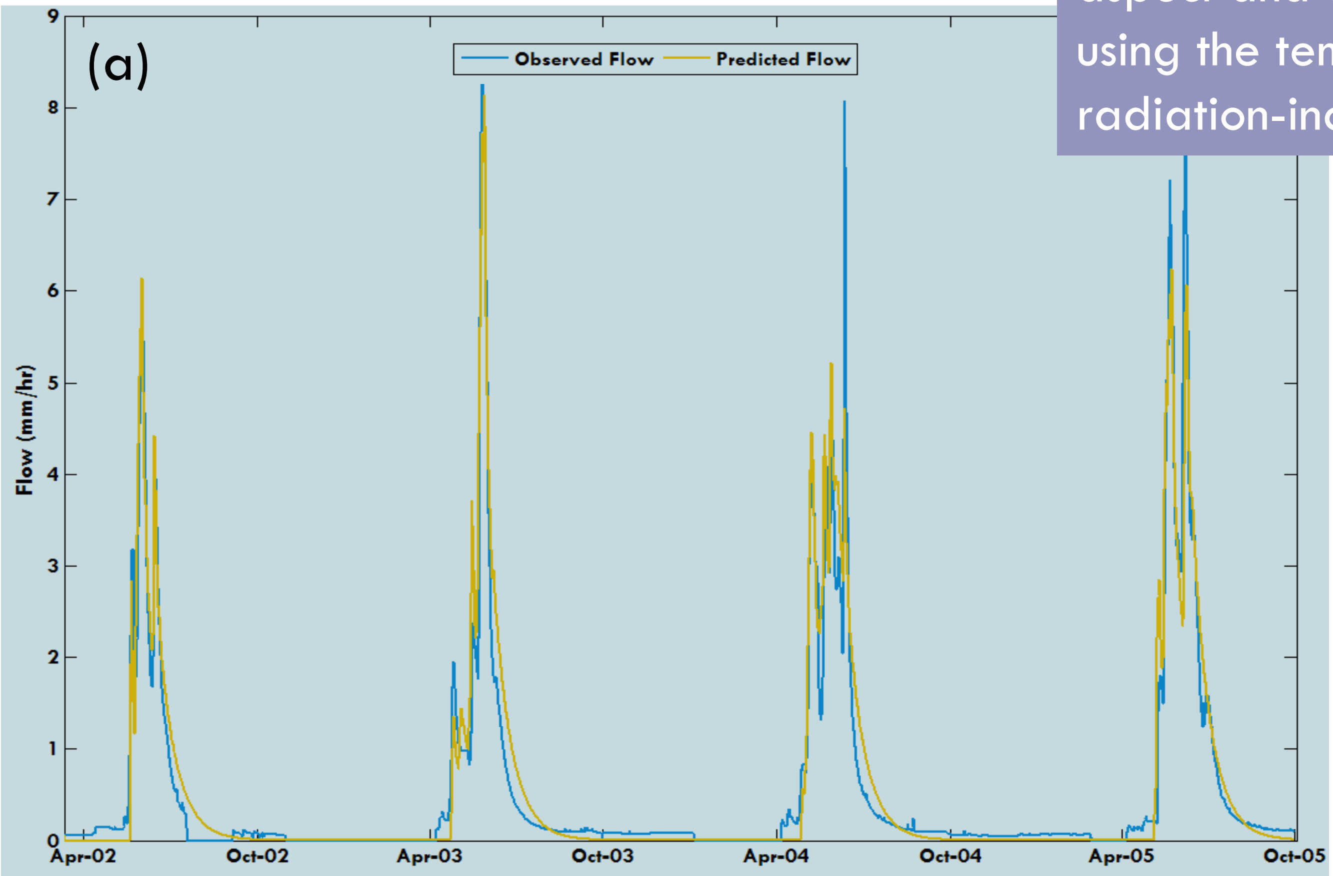




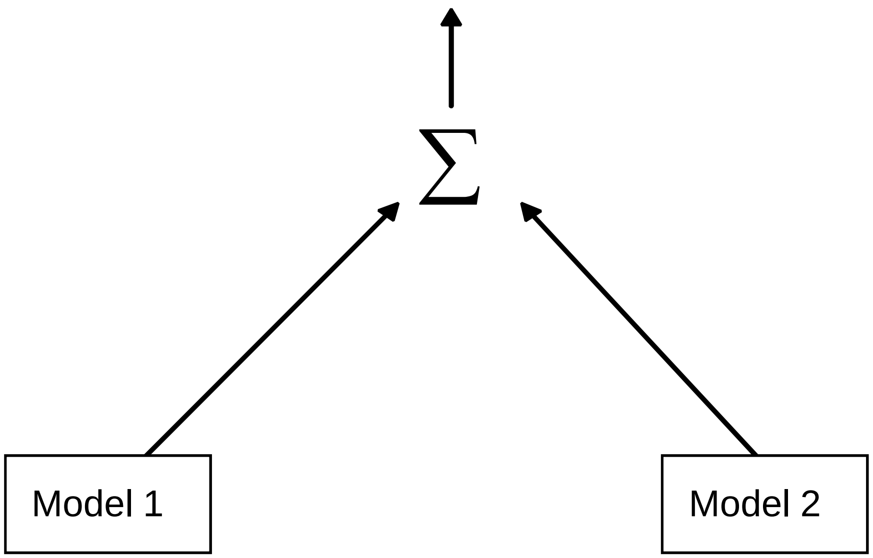
- A solution is provided by an adaptive MCMC algorithm using the history of the sampled parameter states

Inference results

Bucket model semi-distributed by aspect and accounting for snowmelt using the temperature- and radiation-index approach



Assessing model uncertainty via Bayesian Model Averaging



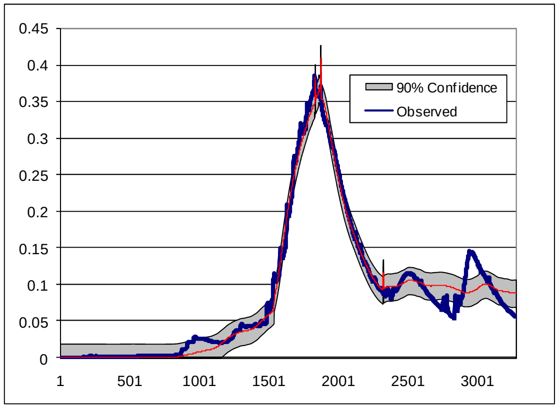
$$m(y \mid M_1) = \int f(y \mid \theta, M_1)\, p(\theta \mid M_1)\, d\theta$$

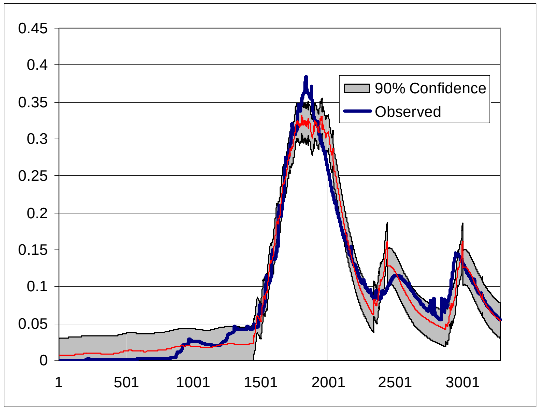
$$P(M_1 \mid y) \propto P(M_1) \bullet m(y \mid M_1)$$

- Probabilistically weight each model
- Ensemble of models is an increasingly accepted way of representing model 'structural' uncertainty
- The Bayesian approach accounts for multiple sources of uncertainty

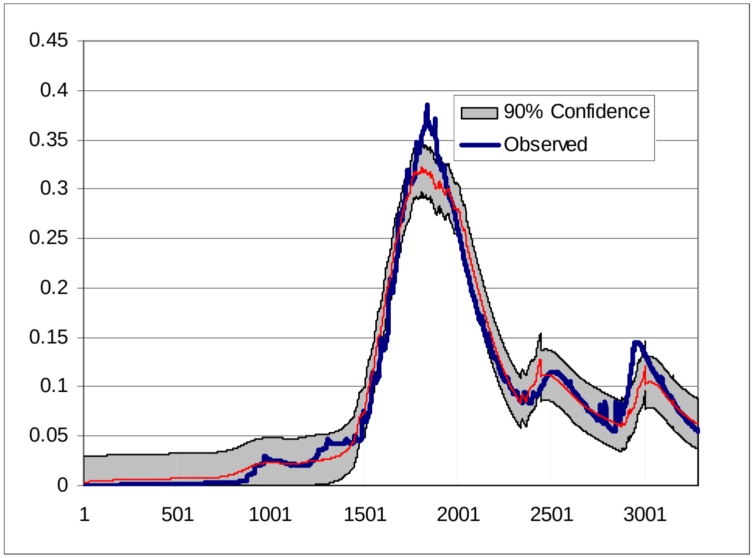
The utility of multi model ensembles

- Models represent competing 'hypotheses' about the first order processes
- Both models provide information on the processes occurring so that the data is better captured





Simple Average of Two Models



Hierarchical Mixtures of Experts

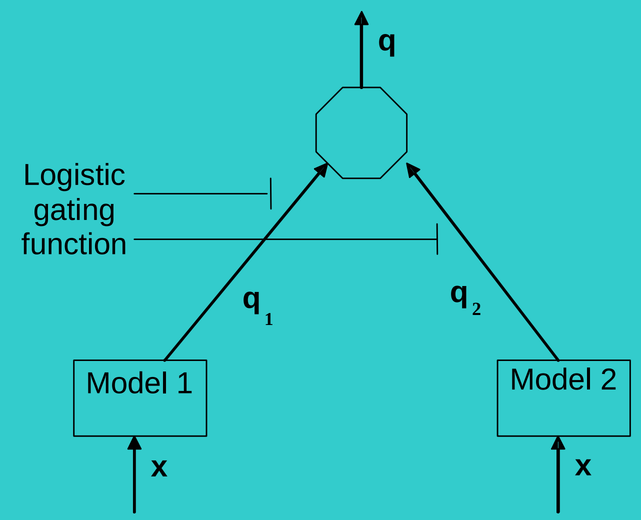
Each conceptual model can be cast as:

$$Q_t = f_{i,t}(x_t;\theta_i) + \varepsilon_{i,t}(\sigma_i^{\,2})$$

The probability of selecting individual models is based on the gating function, using catchment predictors X_t:

$$g_{t,1} = \frac{e^{G(X_t,\beta)}}{1+e^{G(X_t,\beta)}}$$

.

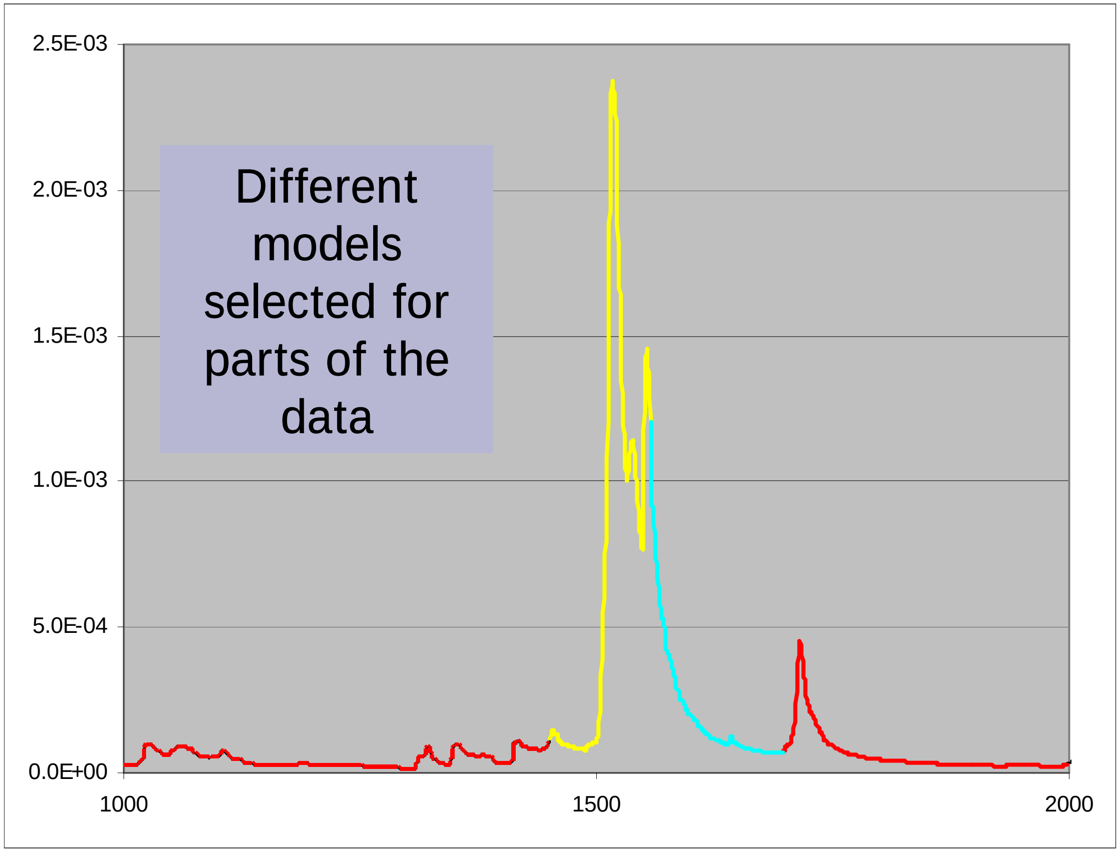


A single-level two-component Hierarchical Mixture of Experts model

Models are sampled via a conditional simulation of independent Bernoulli random variables z_t, with probability specified as:

$$p(z_{t,1}=1\,|\,Q_t,\beta,\theta,\sigma^2) = \frac{p(z_{t,1}=1\,|\,\beta,\theta,\sigma^2)\,P(Q_t\,|\,z_{t,1}=1)}{\sum_{i=1}^{i=2} p(z_{t,i}=1\,|\,\beta,\theta,\sigma^2)\,P(Q_t\,|\,z_{t,i}=1)}$$

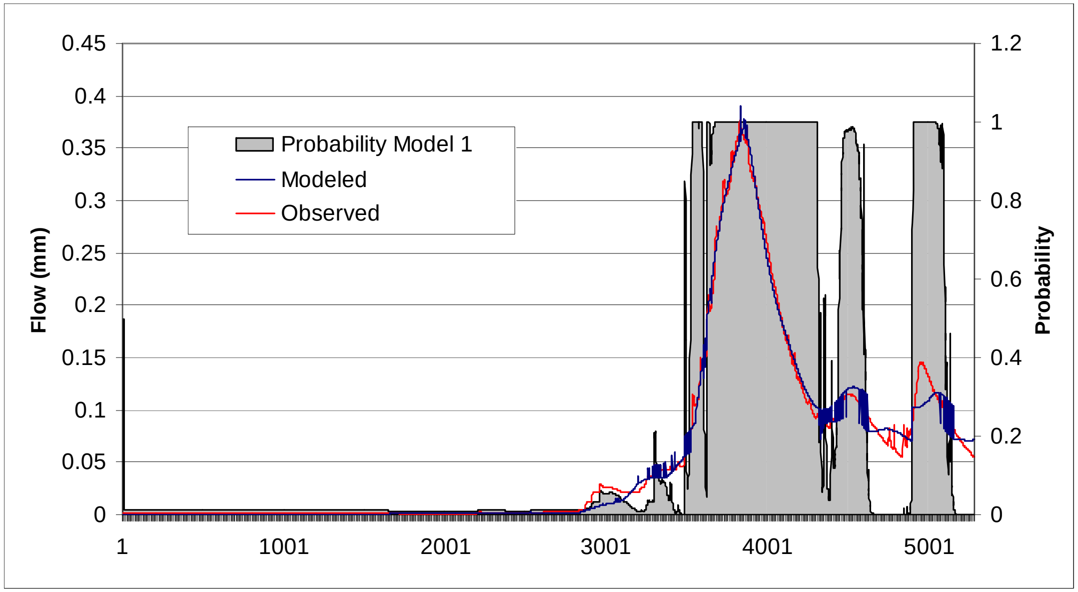
Mixture models- alternative models suitable at different times



- Probabilistically split the data according to some catchment indicators
- Fit separate models to the data and data errors. Models may then ‘specialize’
- Can be likened to Bayesian Model Averaging, where the weights vary in time

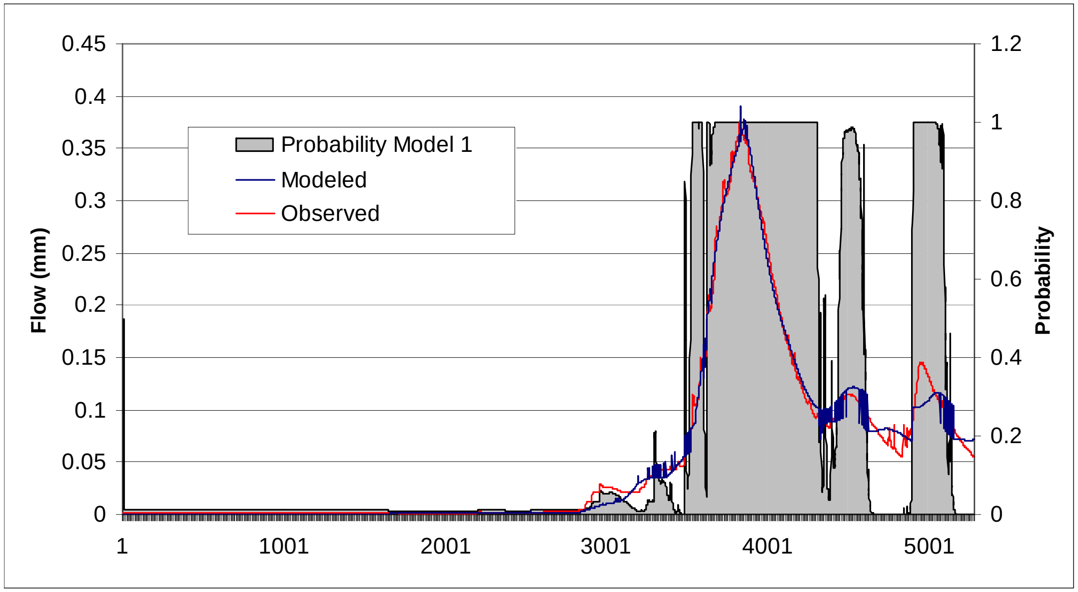
Can fit same model structure with different parameterizations: assumes that model uncertainty does not arise solely out of the assumed model structure

Mixture models- alternative parameterizations suitable at different times



Fit two parameterizations of the single best model (combined temperature/radiation index melt, pdm model)

Mixture models- alternative parameterizations suitable at different times



Model preference changes according to:

•Response to event

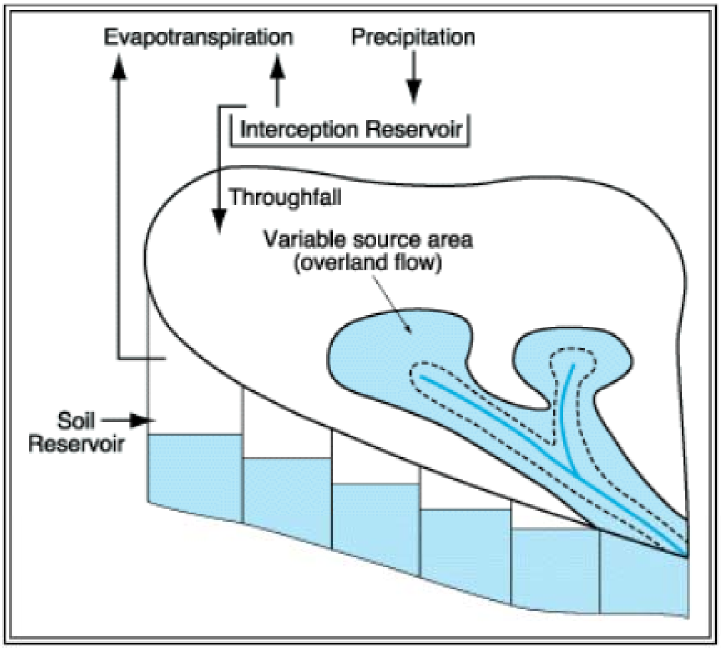
•Time of season

Comparison of alternate model simulations can indicate which parameters are most sensitive to selected calibration period

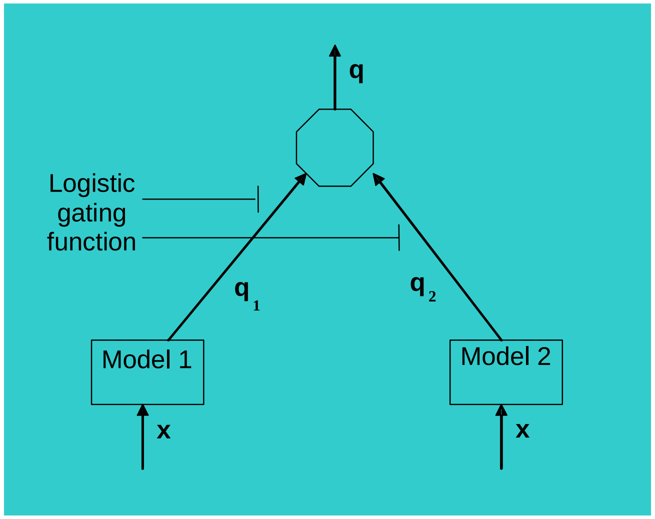
HME approach gives good fit to data, but has problems with:
•Identifiability
•Interpretation
•Predictions

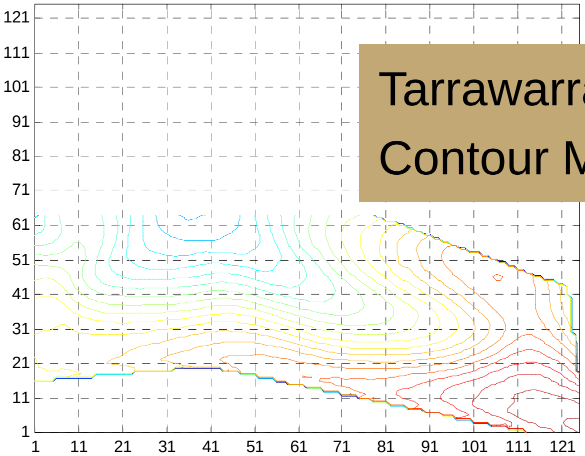
Combining multiple model parameterizations: catchment "states"

Hydrologic model: Topmodel



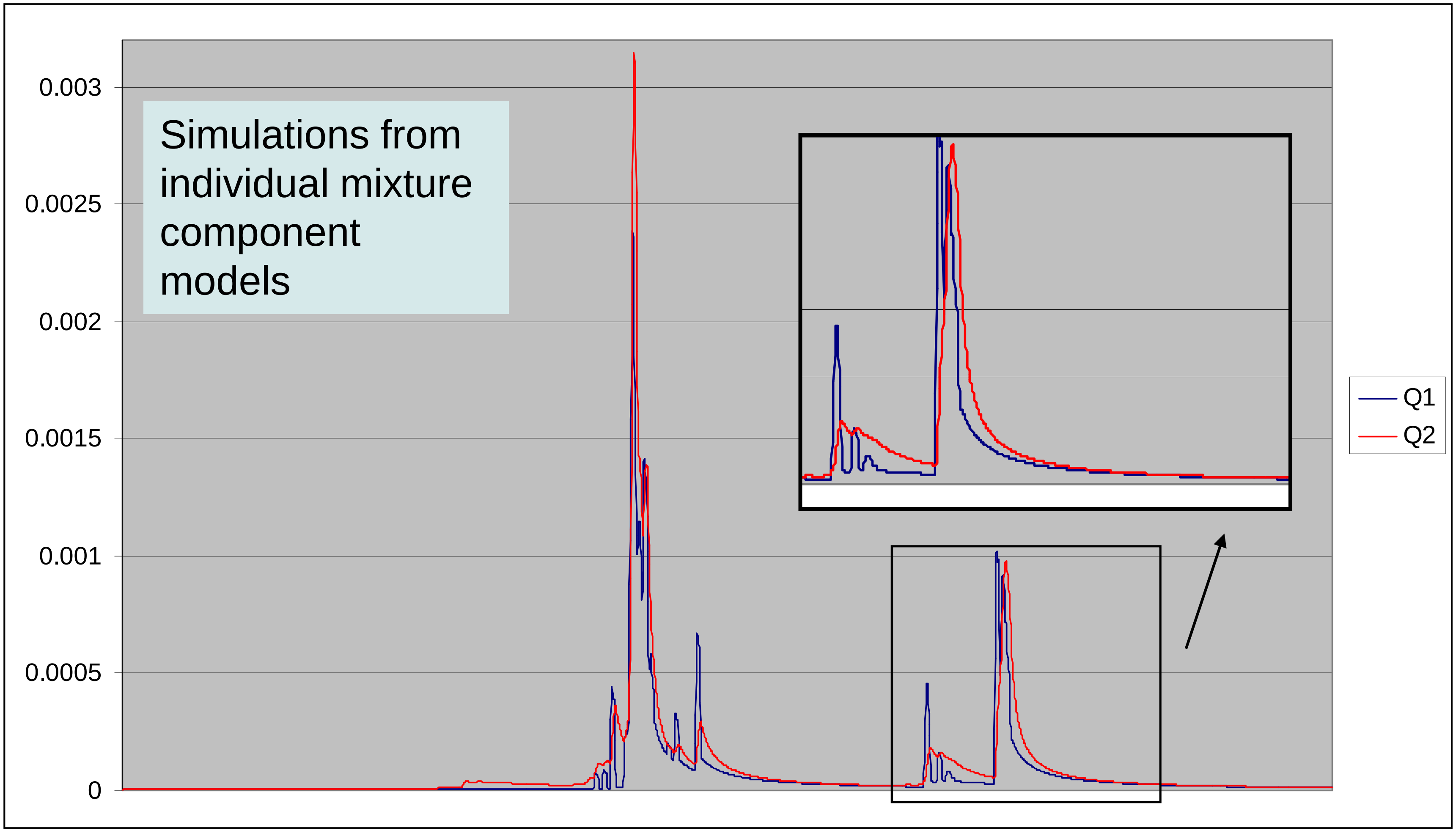
From Hornberger, 1998



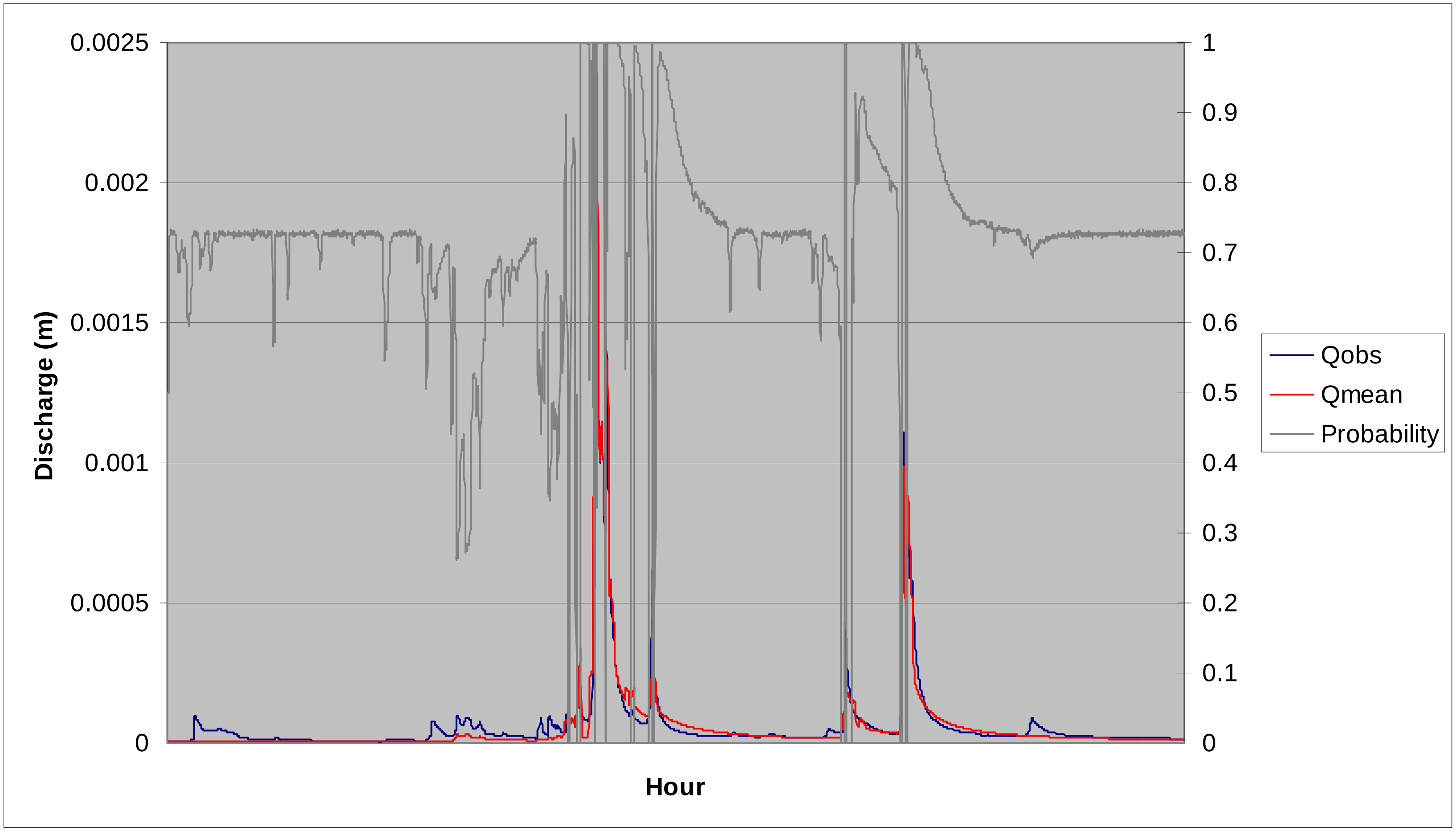


Tarrawarra Catchment Contour Map

Combining multiple model parameterizations: catchment "states"



Combining Multiple Model Parameterizations: Model "States"



What about prediction?

- To use the model for prediction means finding an appropriate catchment descriptor and a function relating this to the probability switching between models
- Possible predictors
  - Antecedent rainfall
  - Modelled catchment storage
  - Time of the year
- The best predictors are often related to the most dynamic catchment mechanisms

Model Aggregation as a Predictive Tool- Comparison of predictors

Model	Predictor	-0.5 BIC
Topmodel	N/A	32685

Model Aggregation as a Predictive Tool- Comparison of predictors

Model	**Predictor**	**-0.5 BIC**
Topmodel	N/A	32685
2 Component HME	Preceding rainfall	33445
	Change in storage deficit	33487
	Change in unsaturated zone storage	33428
	Unsaturated zone storage	33455

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2 Component HME	Preceding rainfall	33445
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Benefits of the HME approach

- HME provides an improved framework for incorporating multiple sources of model uncertainty in hydrology

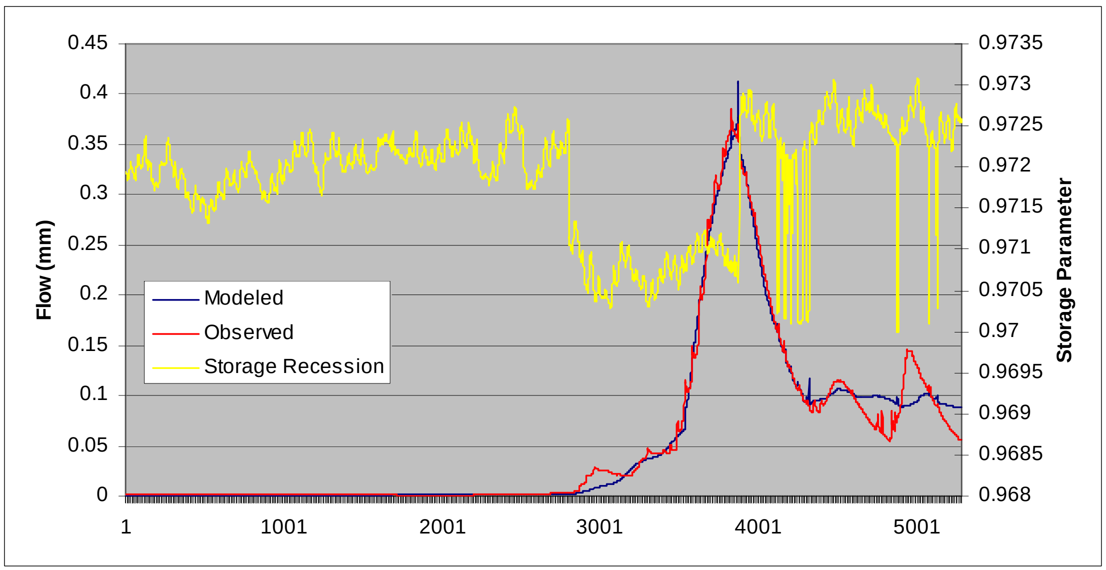
- The HME approach allows combination of multiple models and parameterizations in a single framework

Using Mixture Modeling as a Method of Comparing Model Structures, Parameters and Errors

- HME can highlight problems in the model structure
  - For conceptual models: different responses in wet and dry periods; different ways to model the catchment storage
  - For distributed models: different patterns of soil moisture in wet and dry periods; different assumptions about the recession properties
- A mixture of error distributions can provide better prediction limits and better model heteroscedasticity

Alternative approach: hierarchical model

- Most temporally sensitive parameters are conditioned on observed/modeled exogenous data
- Easier to interpret in light of the conceptualized hydrologic processes
- Look at extent to which parametric variability informs model structural uncertainty



- Storage parameter differentiates alternative HME components
- Condition this on the watershed melt and temperature

Model	Max log-likelihood
Hierarchical	15791
HME	18068

Comparison of aggregation and hierarchical approaches

- How do these approaches compare for:

1. *Assessing model structural uncertainty*
   - Ensemble methods span the breadth of model space with varying degrees to give a better assessment of model uncertainty.
   - HME and hierarchical formulations can highlight problems in the assumed model structure.
2. *Improving model predictions*
   - The ensemble approach should give a more consistent performance for the main variable of interest. The hierarchical approach does hold promise in improving model performance.
3. *Interpretability*
   - Ensemble and HME approaches are less useful beyond the variable of interest. A more complex hierarchical model may give a model structure greater flexibility and a simulation more consistent with internal watershed processes.

Can we use multi-model approaches for better model building?

- For improved conceptual model assessment we should consider that parameter variability and model structural uncertainty are linked.
- The HME approach and multi-model approaches can be used to determine the utility of alternative models under different watershed conditions
- These approaches can be used to improve existing models for better interpretability of internal watershed dynamics and their variability

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Department of Land Resources and Environmental Sciences
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Email: lmarshall@montana.edu

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Adaptive Bayesian algorithms

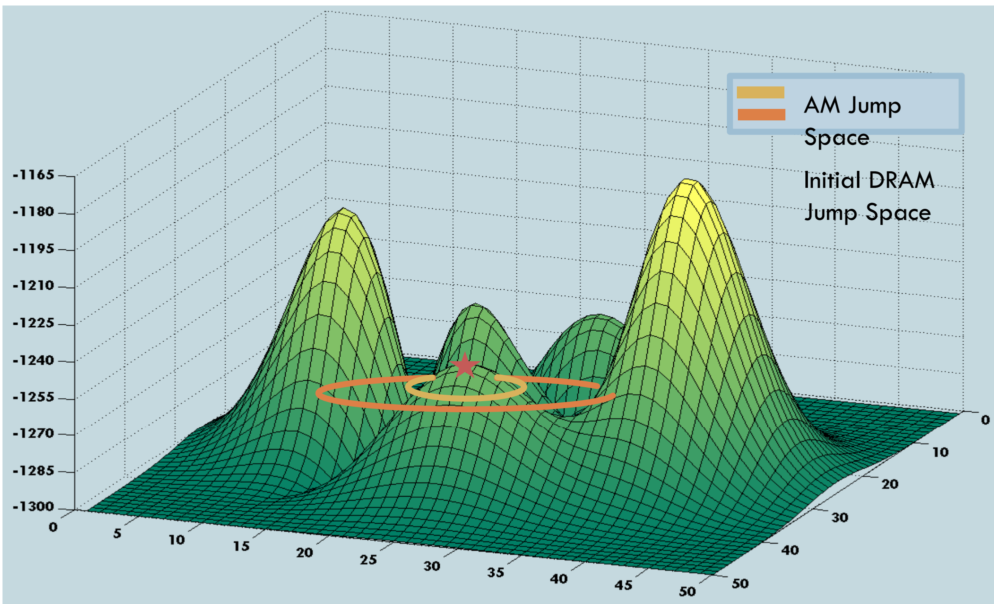

Figure 2. A theoretical parameter surface, diagramming AM & DRAM algorithms' ability to explore the parameter surface. Rings represent distance from the current location an algorithm can explore. These exploration limits illustrate DRAM's ability to search more space and AM's tendency to falsely converge to local maxima because of its more constricted search area.

- The **Adaptive Metropolis (AM)** algorithm (*Haario et al.,* 2001):
  - The covariance of the proposal distribution is updated using the information gained from the simulation thus far.
  - Often plagued by initialization problems, causing the algorithm to become trapped in local optima.
- The **Delayed Rejection Adaptive Metropolis (DRAM)** algorithm (*Haario et al.* 2006):
  - Reduces the probability that the algorithm will remain at the current state.