Exercises

3. Let $u(\cdot)$ be harmonic in the entire space. Show that if $u(\cdot)$ is bounded then it is necessarily a constant. Hint: Assume $u \leq M$ and apply Harnack’s inequality to $M - u$ at $\xi \in S_R(0)$ where $|\xi| < R$. Then let $R \to \infty$.

4. Show that a consequence of the two theorems on page 17 of the Notes is that the Dirichlet boundary value problem

\[-\Delta u(x) = f(x), \quad x \in G,\]
\[u(s) = g(s), \quad s \in \partial G,\]

has a unique solution for each $f \in C^1(\bar{G})$ and $g \in C(\partial G)$.