1. (a) Show that there is more than one solution of the initial-value problem

\[ \dot{u}(t) = u^\alpha(t), \quad u(0) = 0, \]

on \( t \geq 0 \), where the number \( \alpha \) satisfies \( 0 < \alpha < 1 \).

(b) Show that there is at most one solution of the initial-value problem

\[ \dot{u}(t) = -u^{1/3}(t), \quad u(0) = u_0, \]

on \( t \geq 0 \).

2. (a) Show that if the function \( f(t, s) \) satisfies

\[ (f(t, u) - f(t, v))(u - v) \leq K(u - v)^2; \tag{1} \]

then there is at most one solution of the initial-value problem

\[ \dot{u}(t) = f(t, u(t)), \quad u(0) = u_0, \]

on \( t \geq 0 \).

(b) Describe in terms of monotonicity properties the functions \( f(t, u) \) for which the condition (1) holds.

(c) Show that if

\[ |f(t, u) - f(t, v)| \leq K|u - v|; \tag{2} \]

for some constant \( K \), then there is at most one solution of the initial-value problem

\[ \dot{u}(t) = f(t, u(t)), \quad u(0) = u_0, \]

on any interval of the form \( |t| \leq T \).

(d) Describe in terms of growth-rate properties the functions \( f(t, u) \) for which the condition (2) holds.

3. Let \( u(t) \) and \( v(t) \) be solutions of the initial value problems

\[ \dot{u}(t) + \sin(u(t)) = 0, \quad u(0) = u_0, \]

\[ \dot{v}(t) + \sin(v(t)) + \frac{1}{10} \sin(10v(t)) = 0, \quad u(0) = u_0. \]

Estimate the difference \( |u(t) - v(t)| \) for \( 0 \leq t \leq T \) in terms of \( u_0 - v_0 \).