1. For Lebesgue measure on \( X = Y = [a, b] \), show that each open set in \( X \times Y \) is measurable.

2. Let \( g(\cdot) \) be integrable on the measure space \((X, \mu_1)\) and \( h(\cdot) \) be integrable on the measure space \((Y, \mu_2)\). Define \( f(x, y) = g(x)h(y) \) for \((x, y) \in X \times Y\). Show that \( f \) is integrable on \((X \times Y, \mu)\) with \( \mu = \mu_1 \times \mu_2 \) and that

\[
\int_{X \times Y} f \, d\mu = \int_X g \, d\mu_1 \int_Y h \, d\mu_2.
\]

(Do not assume \( \sigma \)-finiteness of the measure spaces.)

3. Let \( X = Y \) be positive integers with the counting measure, and define the function on \( X \times Y \) by

\[
f(x, y) = \begin{cases} 
2 - 2^{-x} & \text{if } x = y, \\
-2 + 2^{-x} & \text{if } x = y + 1, \\
0 & \text{otherwise.}
\end{cases}
\]

Evaluate each of the following:

(a) \( \int_X f(\cdot, y) \, d\mu_1 \)

(b) \( \int_Y f(x, \cdot) \, d\mu_2 \)

(c) \( \int_X \int_Y f \, d\mu_2 \, d\mu_1 \)

(d) \( \int_Y \int_X f \, d\mu_1 \, d\mu_2 \)

(e) \( \int_{X \times Y} f \, d\mu \)

4. Show that the existence of the integral

\[
\int_X \left( \int_Y |f(x, y)| \, d\mu_2 \right) \, d\mu_1
\]

implies the existence of

\[
\int_{X \times Y} |f(x, y)| \, d(\mu_1 \times \mu_2).
\]

(This is Problem 6 on page 361 of Kolmogorov - Fomin. See the Hint given there.)