Math Faculty as Partners in Teaching a non-Euclidean Geometry Course for K-12 Teachers

David Damcke, Tevian Dray, Maria Fung, Dianne Hart, and Lyn Riverstone
Summary of Partnership

- Curriculum development
- Course implementation
- Course expansion and revision
- Faculty professional development and growth
- Dissemination efforts
Geometry for K-12 Teachers

- Oregon Mathematics Leadership Institute (OMLI)
  - NSF-funded partnership project—OSU/PSU/TDG/10 OR school districts (NSF/EHR-0412553; ODE/Oregon ESEA Title II-B MSP)
  - One of 6 mathematics courses
  - Aimed at deepening geometry content knowledge of K-12 teachers
Non-Euclidean Geometry

- OHLA district leaders recommend non-Euclidean geometry
  - Spherical geometry
  - Taxicab geometry
- “Comparing Different Geometries”
- Intense: 15 two-hour sessions
- Team-taught
The Geometry Team

- Research mathematician with interest in mathematics education
- Master teacher
- Mathematics education specialist from a teaching university
- 2 college instructors with varied teaching experience
Curriculum Highlights

- Unit on spherical geometry
  - Lines
  - Parallel and perpendicular lines
  - Set of points equidistant from 2 different points
  - Common perpendicular
  - Polygons: triangles, “squares”
  - Circles
Curriculum Highlights Continued

- Unit on Taxicab geometry
  - Distance
  - Midpoints
  - Sets of points equidistant from two given points
  - Squares
  - Circles and $\pi$
  - Triangles and congruence
Preparation for Delivery

Teachers Development Group workshops and readings:
- “Best Practices”
- Cooperative Learning
- Promoting Discourse in the Classroom

- Lesson plans
- Daily debrief sessions
- Reflection and revision
OMLI’s Role

- Numerous opportunities to meet
  - Year-round communication
  - Retreats
- Facilitation of exchange of ideas
  - In person
  - Via e-mail
- Development of group dynamics
  - Cooperative division of labor
  - Embracing similarities and differences in style
Implementation

- Building comparison charts
  - Spherical with Euclidean geometry
  - Taxicab with Euclidean geometry

- Group projects
  - Capstone experience: extension of ides
  - Presentations and posters

Equilateral Triangles
Pedagogical Lessons

- More and better mathematics discourse and learning:
  - Rich mathematical tasks
  - Orchestrated cooperative learning
  - Effective facilitation
  - Comparison: relating, connecting, multiple representations
Expansion and Revision

- Focus on undefined terms, axioms, and precise definitions
- Switch of topic order and introduce new manipulative aids: Etch-a-sketch
- More sophisticated cooperative learning strategies: jigsaw puzzle, differentiated tasks
- Role of writing: from writing reflections, to prompts, to exit cards
Faculty Growth

- Challenges and triumphs of team teaching
  - Prepare, share, collaborate
  - Draw on each other’s strengths
  - Stay flexible
  - Learn and reflect
- Lesson study
- Open door policy
Faculty Impact

- Threefold impact on own classrooms
  - Mathematics discourse
    - Mathematical tasks
    - Group work
    - Facilitation skills
  - Projects: capstone experiences
  - Assessment: reflective, peer, rubrics
Mathematical Tasks

- Multiple entry points
- Group-worthy
- Applied, hands-on nature
- More open ended

- Use an Etch-a-Sketch to explore the notion of Taxicab geometry distance by measuring the length of the diagonal line on the Etch-a-Sketch by turning one knob at a time.
Group Work

- Awareness of equity issues
  - Heterogeneous groups
  - Careful selection and rotation
  - Roles

- Judicious Orchestration
  - Size of groups
  - Huddles
  - Protocols

- Advanced collaborative learning strategies
  - Jigsaw
Facilitation Skills

- Better questioning techniques
- Observing and sequencing
  - from simple to complex
  - from kernels of idea to its development
  - handling student mistakes by letting groups respond
- Connecting student ideas better
Focus on Students’ Thinking

☐ “Less is more.”
☐ Listen, observe, take notes.
☐ Use student feedback to adjust teaching.
☐ Focus on justification and sense-making at all times.
Dissemination Efforts

- Entire set of curriculum materials is ready for publication through *Teachers Development Group*
- Article accepted in The Montana Mathematics Enthusiasts Monograph on Discourse (pedagogy)
- Article in preparation to be submitted to School Science and Mathematics (content)
- “Dare to Compare? Introduction to Using Comparison Charts in the Mathematics Classroom” appeared in May/June 2008 TOMT