$\mathfrak{e}_{8(-24)}$

and the Standard Model

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Joint work with

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Our Recent Work

- Wilson, Dray, & Manogue (IIG 2023) [arXiv:2204.04996]
- Manogue, Dray, & Wilson (JMP 2022) [arXiv:2204.05310]
- Dray, Manogue, & Wilson (submitted) [arXiv:2309.00078]
- Dray, Manogue, & Wilson (in preparation)

  "A New Division Algebra Representation of E7"

Our Earlier Related Papers

Fairlie & Manogue (PRD 1986, PRD 1987), Manogue & Sudbery (PRD 1989), Schray (PhD 1994), Manogue & Schray (JMP 1993), Dray & Manogue (AACA 1998, CPC 1998, IJTP 1999), Manogue & Dray (MPLA 1999), Dray, Janesky, & Manogue (AACA 2000), Dray, Manogue, & Okubo (AGG 2002), Dray & Manogue (CAA 2000, CMUC 2010), Manogue & Dray (JPhys 2010), Wangberg (PhD 2007), Wangberg & Dray (JMP 2013, JAA 2014), Dray, Manogue, & Wilson (CMUC 2014), Kincaid (MS 2012), Kincaid and Dray (MPLA 2014), Dray, Huerta, & Kincaid (LMP 2014)

Short List of References

- Jordan (1933), Jordan, von Neumann, & Wigner (1934), Freudenthal (1954, 1964), Tits (1966), Vinberg (1966), Günaydin & Gürsey (1973, 1974), Gürsey, Ramond, & Sikivie (1976), Bars & Günaydin (1980), Olive & West (1983), Kugo & Townsend (1983), Chung & Sudbery (1987), Goddard, Nahm, Olive & Ruegg (1987), Corrigan & Hollowood (1988), Dixon (1994), Okubo (1995), Günaydin, Koepsell, & Nicolai (2001), Barton & Sudbery (2003), Cederwall (2007), Lisi (2007, 2010), Baez & Huerta (2010), Chester, Marrani, & Rios (2021), Furey (2015), Furey & Hughes (2022ab), Marrani, Corradetti, Chester, Aschheim, & Irwin (2022), Chester, Marrani, Corradetti, Aschheim, & Irwin (2023). (Complete references in our recent papers.)
- Also see the wonderful list of references and history in the talk 2 weeks ago by David Chester.

Lorentz + Internal

- All particles have both:
  - Lorentz properties:
    - Bosons: spin 1 (photon, gluons, $W^{\pm}, Z$)
    - Bosons: spin-0 (Higgs)
    - Fermions: spin-1/2 (leptons and quarks)
  - Internal symmetry properties: charge, color,
- Coleman-Mandula says that this can only be a tensor product.

Dirac Spinors and Chirality

- All fermions are solutions of the Dirac equation.
  $(p_\mu \gamma^\mu - m)\ \psi = 0$
- ψ built of two spin $1/2$ reps of Lorentz with opposite chirality
- The weak force $\mathfrak{su}(2)$ acts on only one chirality.

The Standard Model

$\mathbf{Fermions\ (Dirac\ Spinors)}$		$\mathbf{Bosons\ (Vectors)}$	
Leptons		Mediators	
$e^-,\ \mu^-,\ \tau^-$	$\text{charge} = -1$	γ	$\mathfrak{u}(1)$
$\mu_e,\ \mu_\mu,\ \mu_\tau$	$\text{charge} = 0$	$W^\pm,\ Z$	$\mathfrak{su}(2)$
Quarks			
$u,\ c,\ t$	$\text{charge} = \frac{2}{3}$	gluons	$\mathfrak{su}(3)$
$d,\ s,\ b$	$\text{charge} = -\frac{1}{3}$		
		Higgs (Scalar)	

- Particles and Anti-particles
- Generations: 3 copies of fermions differing only by mass

GUTs

- Is there a (semi-)simple group that contains $\mathfrak{su}(3) \times \mathfrak{su}(2) \times \mathfrak{u}(1)$?
- Common candidates are $\mathfrak{su}(5)$ and $\mathfrak{so}(10)$

Labels

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}'$	$\{1, U\}$	$\{\ldots, k\}$	$\{\ldots, i, j\}$	$\{\ldots, i\ell, j\ell, k\ell, \ell\}$
$\mathbb{C}'$	$\{\ldots, L\}$			
$\mathbb{H}'$	$\{\ldots, K, KL\}$			
$\mathbb{O}'$	$\{\ldots, I, IL, J, JL\}$			

New Description of $\mathfrak{e}_8$
(Wilson et al.)

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}'$	$\mathfrak{so}(3)$	$\mathfrak{su}(3)$	$\mathfrak{su}(3,\mathbb{H})$	$\mathfrak{f}_4$
$\mathbb{C}'$	$\mathfrak{sl}(3,\mathbb{R})$	$\mathfrak{sl}(3,\mathbb{C})$	$\mathfrak{sl}(3,\mathbb{H})$	$\mathfrak{e}_{6(-26)}$
$\mathbb{H}'$	$\mathfrak{sp}(6,\mathbb{R})$	$\mathfrak{su}(3,3)$	$\mathfrak{d}_{6(-6)}$	$\mathfrak{e}_{7(-25)}$
$\mathbb{O}'$	$\mathfrak{f}_{4(4)}$	$\mathfrak{e}_{6(2)}$	$\mathfrak{e}_{7(-5)}$	$\mathfrak{e}_{8(-24)}$

- Everything is 3x3 “matrices” with two “labels”
- Ordinary matrices/commutators in quaternionic cases.
- Generalize commutators for double-labeled diagonal elements.

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}'$	$\mathfrak{so}(2)$	$\mathfrak{so}(3)$	$\mathfrak{so}(5)$	$\mathfrak{so}(9)$
$\mathbb{C}'$	$\mathfrak{so}(2,1)$	$\mathfrak{so}(3,1)$	$\mathfrak{so}(5,1)$	$\mathfrak{so}(9,1)$
$\mathbb{H}'$	$\mathfrak{so}(3,2)$	$\mathfrak{so}(4,2)$	$\mathfrak{so}(6,2)$	$\mathfrak{so}(10,2)$
$\mathbb{O}'$	$\mathfrak{so}(5,4)$	$\mathfrak{so}(6,4)$	$\mathfrak{so}(8,4)$	$\mathfrak{so}(12,4)$

Orthogonal Lie Algebras

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}'$	$\mathfrak{so}(3)$	$\mathfrak{su}(3)$	$\mathfrak{su}(3,\mathbb{H})$	$\mathfrak{f}_4$
$\mathbb{C}'$	$\mathfrak{sl}(3,\mathbb{R})$	$\mathfrak{sl}(3,\mathbb{C})$	$\mathfrak{sl}(3,\mathbb{H})$	$\mathfrak{e}_{6(-26)}$
$\mathbb{H}'$	$\mathfrak{sp}(6,\mathbb{R})$	$\mathfrak{su}(3,3)$	$\mathfrak{d}_{6(-6)}$	$\mathfrak{e}_{7(-25)}$
$\mathbb{O}'$	$\mathfrak{f}_{4(4)}$	$\mathfrak{e}_{6(2)}$	$\mathfrak{e}_{7(-5)}$	$\mathfrak{e}_{8(-24)}$

+Spinors

- 2x2 Lie algebras are degree 2 in Clifford algebra.
- 2x2->3x3 adds spinor representations with appropriate Bott periodicity.

Type Structure

$$\begin{pmatrix} D & X & -Z^\dagger \\ -X^\dagger & \pm D & Y \\ Z & -Y^\dagger & 0 \end{pmatrix}$$

- *D*s must have both labels in the same division algebra. (We don't always write {1, *U*}).
- *X*s, *Y*s, *Z*s have one label in each division algebra.

Choices for Octions Models

- Everything in real $\mathfrak{e}_8$
  - The minimal representation is the adjoint, so actors and actees are in the same space.
  - Don’t complexify, pay attention to signature.
  - Always stay in the Magic Square.
- Prioritize Lorentz, weak, color over generation.
- No gravity.
- Allow Clifford and Jordan algebra structures to emerge from $\mathfrak{e}_8$.

Rules of Today's Game

- Pick an entry in the magic square.
- Assign division algebra labels.
- Decompose into smaller entry and centralizer.
- Interpret *D*s and *X*s as adjoints/bosonic reps.
- Interpret the *Y*s and *Z*s as fermionic reps.

Example: $\mathfrak{a}_{5(-7)}$

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}'$	$\{1, U\}$	$\{\ldots, k\}$	$\{\ldots, i, j\}$	$\{\ldots, i\ell, j\ell, k\ell, \ell\}$
$\mathbb{C}'$	$\{\ldots, L\}$		$\mathfrak{a}_{5(-7)}$	
$\mathbb{H}'$	$\{\ldots, K, KL\}$			
$\mathbb{O}'$	$\{\ldots, I, IL, J, JL\}$			

Content of: $\mathfrak{a}_{5(-7)}$

- Adjoint $\mathfrak{so}(5,1)$ (labels $\{U, L, 1, i, j, k\}$)
- Adjoint $\mathfrak{su}(2)$

$$GS_k = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k \end{pmatrix}$$

- Adjoint $\mathfrak{so}(1,1)$

$$S_L = \begin{pmatrix} L & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & -2L \end{pmatrix}$$

- 2 spinor 8s of $\mathfrak{so}(5,1)$ (labels $\{U \pm L, 1, i, j, k\}$)

Extension: $\mathfrak{e}_{6(-26)}$

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}'$	$\{1, U\}$	$\{\ldots, k\}$	$\{\ldots, i, j\}$	$\{\ldots, i\ell, j\ell, k\ell, \ell\}$
$\mathbb{C}'$	$\{\ldots, L\}$		$\mathfrak{a}_{5(-7)}$	$\mathfrak{e}_{6(-26)}$
$\mathbb{H}'$	$\{\ldots, K, KL\}$			
$\mathbb{O}'$	$\{\ldots, I, IL, J, JL\}$			

Content of: $\mathfrak{e}_{6(-26)}$

- Add labels in $\mathbb{H}_\perp$ i.e. $\{i\ell, j\ell, k\ell, \ell\}$
- $\mathfrak{e}_6 = \mathfrak{a}_5 \oplus \mathfrak{su}(2) \oplus 40$
- Lorentz structures inside representations of $\mathfrak{a}_5$
  - 4 more $\mathfrak{so}(5,1)$ Lorentz vectors with labels in $\mathfrak{so}(4) = \mathfrak{su}(2) \oplus \mathfrak{su}(2)$
  - Double the number of spinors (16) by including those with labels in $\mathbb{H}_\perp$

Rewrite Quaternionic Matrices

$$GS_k = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k \end{pmatrix}$$

$$D_{\ell,k\ell} - D_{i\ell,j\ell}$$

$$D_{\ell,i\ell} - D_{j\ell,k\ell}$$

$$D_{\ell,j\ell} - D_{k\ell,i\ell}$$

$\mathfrak{so}(4) = \mathfrak{su}(2) \oplus \mathfrak{su}(2)$

Old	New
$D_{\ell,k\ell} - D_{i\ell,j\ell}$	$D_{\ell,k\ell} + D_{i\ell,j\ell}$
$D_{\ell,i\ell} - D_{j\ell,k\ell}$	$D_{\ell,i\ell} + D_{j\ell,k\ell}$
$D_{\ell,j\ell} - D_{k\ell,i\ell}$	$D_{\ell,j\ell} + D_{k\ell,i\ell}$

Handedness

- $\mathfrak{e}_6$ is our first example with octonionic content.
- $\mathfrak{so}(4) = \mathfrak{su}(2) \oplus \mathfrak{su}(2)$ all with $\mathbb{H}_\perp$ labels.
- New $\mathfrak{su}(2)$ annihilates the old spinors.
- Old $\mathfrak{su}(2)$ annihilates the new spinors.
- Second Weyl handedness of $\mathfrak{so}(5,1)$ spinors emerges from non-associativity of octonions

Decompositions

- $\mathfrak{so}(p+q)=\mathfrak{so}(p)\oplus\mathfrak{so}(q)\oplus p\times q$
- The spinors decompose appropriately.

Extension: $\mathfrak{e}_{8(-24)}$

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}'$	$\{1, U\}$	$\{\ldots, k\}$	$\{\ldots, i, j\}$	$\{\ldots, i\ell, j\ell, k\ell, \ell\}$
$\mathbb{C}'$	$\{\ldots, L\}$			
$\mathbb{H}'$	$\{\ldots, K, KL\}$			
$\mathbb{O}'$	$\{\ldots, I, IL, J, JL\}$			$\mathfrak{e}_{8(-24)}$

Content of: $\mathfrak{e}_{8(-24)}$

- $\mathfrak{e}_8 = \mathfrak{e}_6 \oplus \mathfrak{sl}(3,\mathbb{R}) \oplus 27 \times 3 \oplus \overline{27} \times \overline{3}$
- Add Labels $\{I \pm IL, J \pm JL, K \pm KL\}$
- 27 is literally a Jordan algebra times a null label => all the old work about $\mathfrak{e}_6$ acting on Jordan algebras applies straightforwardly.

Take Home Messages

If you break $\mathfrak{e}_8$ in these ways:

- $\mathfrak{a}_5$ gives chirality, right-handed leptons, and $\mathfrak{su}(2)$
- $\mathfrak{e}_6$ adds left-handed leptons, another $\mathfrak{su}(2)$ and potential weak mediators.
- $\mathfrak{e}_8$ adds colored $\left(3+\overline{3}\right)$ Jordan 27s to build quarks/baryons with potentially testable properties.
- $\mathfrak{e}_7$ shows how to build determinants of the 27 representations of $\mathfrak{e}_6$ using color.