Active Engagement: Lessons learned from the Paradigms and Bridge projects

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Get Ready:

Please sit with one or two partners.

For each group, pick up:

- a small whiteboard
- a whiteboard pen
- a high tech eraser (i.e. tissue/napkin)
- a set of colored letters

Teaching

Good teaching is like picking up someone else's baby.

Using the Quaternions to Implement Rotations

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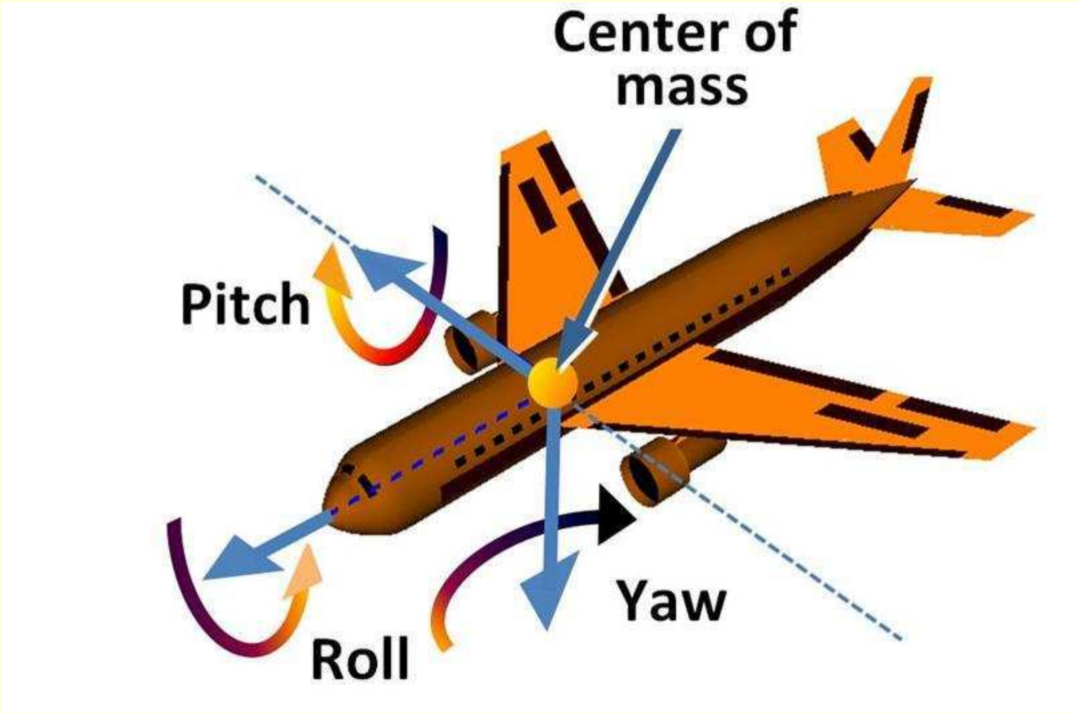
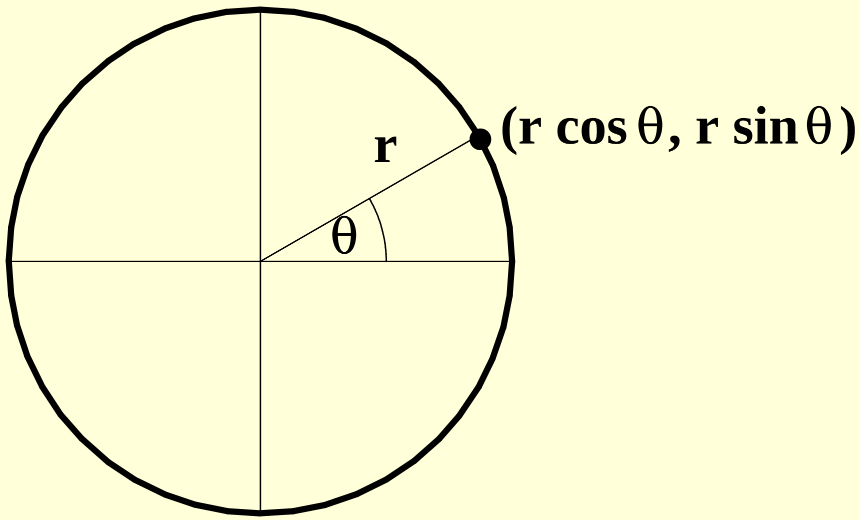
Introduction

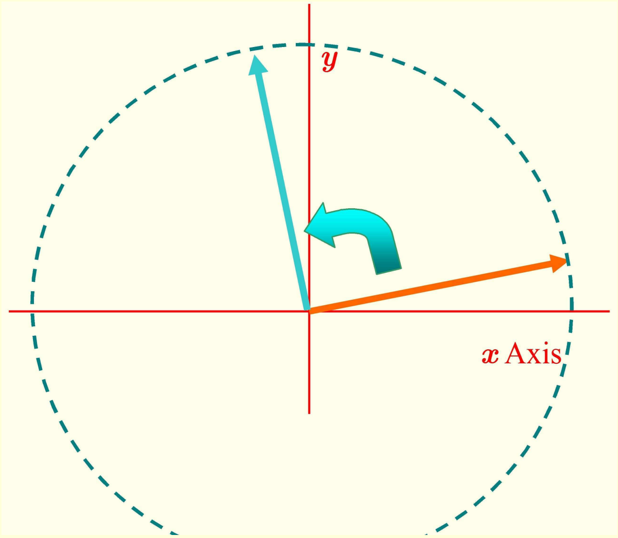


3-d rotations: Aeronautics, robotics, computer graphics, ...

New Content: Use quaternions to implement rotations

Trigonometry





Polar coordinates: $\boxed{x = r\cos\theta;\ y = r\sin\theta.}$

▸ Circle Trig ▸ Rotations

Research-Based Instruction

Things to consider:

- Whenever possible, base your instruction on what is known about incoming student resources.
- Example: Dr. Emily Smith (OSU 2016) showed that many upper-division physics students know triangle trigonometry, but not unit-circle trigonometry. This causes problems with complex numbers.

Classroom implementation:

- Implication: Use the circle simulation.

Simulation/Demo

Things to consider:

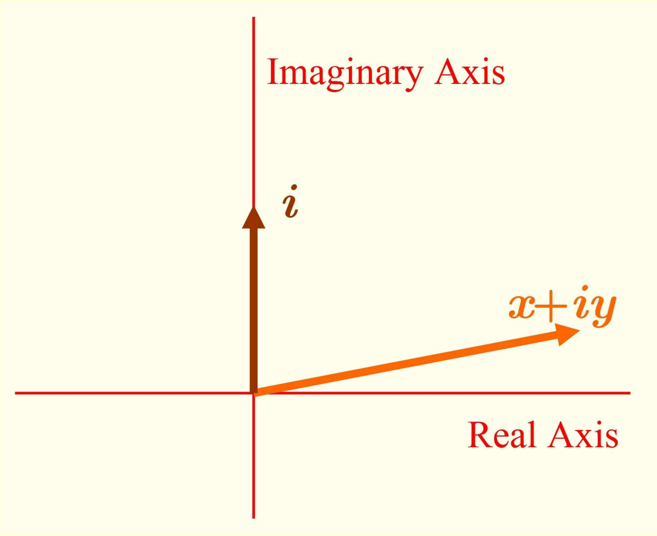
- Decide between black box or open coding.
- Show geometry and/or time dependence.
- Plan specific questions: Students need to be taught to ask relevant questions or to explore parameter space.

Classroom implementation:

- Stand behind students to see if they are having problems with the computer.

Complex Plane

$\mathbb{C} = \mathbb{R} \oplus i\mathbb{R}$



$\boxed{i^2 = -1}$

$$(x, y) \longmapsto x + iy$$

$$x + iy = r \cos\theta + i\, r \sin\theta = r\, e^{i\theta}$$

Special case: $e^{\pm i\pi/2} = \pm i$

$$\boxed{e^{i\pi} + 1 = 0}$$

Representing Complex Numbers

- Please stand up.
- Use left hand.
- Real axis points forward.
- Imaginary axis points upward.

Show me:

- 1
- $2\,i$
- $1 + i$

Kinesthetic Activity

Things to consider:

- Everyone is awake!
- Teacher can see what everyone is thinking.
- Highlights geometric reasoning.
- Students get geometric cues from others.
- Students must make a decision.
- Student can be asked to translate representations.

Classroom Implementation:

- *Please stand up.*
- *Show me ….*
- *Thank you, you can sit down.*

Multiplication by i

$$\boxed{(1+i)i = i-1}$$

If a complex number is multiplied by i, the corresponding vector is:

A: Reflected about the x-axis

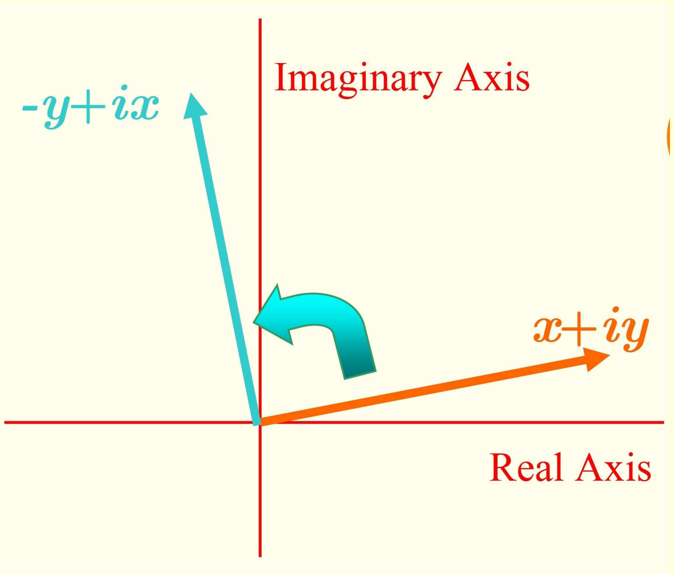
B: Reflected about the y-axis

C: Rotated by $\frac{\pi}{2}$ (90°) counterclockwise

D: Rotated by $\frac{\pi}{2}$ (90°) clockwise

DO NOT VOTE UNTIL TOLD TO DO SO!

Multiplication by i



Multiplication by i: $(x + i\,y)i = i\,x + i^2 y = -y + ix$

Multiplication by $\mathbf{e^{i\theta}}$: $(r\,e^{i\alpha})e^{i\theta} = r\,e^{i(\alpha+\theta)}$

(Rotates counterclockwise by θ!)

Concept Tests/Peer Instruction/Clickers

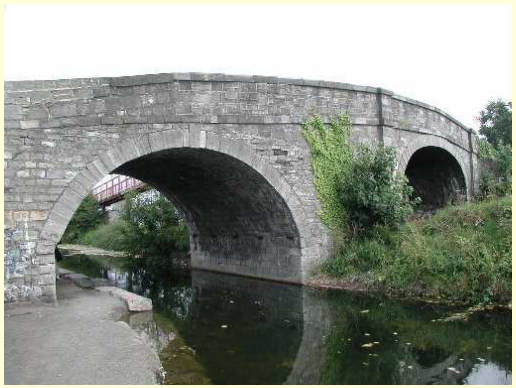
Things to consider:

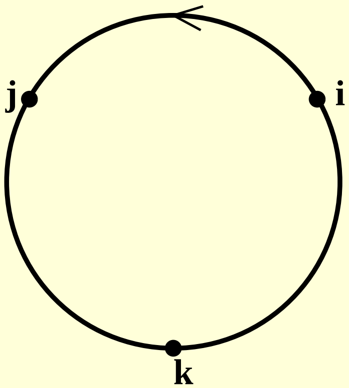
- Asks students to make a commitment.
- Asks students to defend an answer.
- Good questions: conceptual, focus on common mistakes.

Classroom implementation:

- 3 “response” systems: clickers, ABCD cards, (whiteboards).
- Two stages (or gather responses and discuss).
- Simultaneous and anonymous.
- *Convince your neighbor.*

$\mathbb{H} = \mathbb{C} \oplus \mathbb{C}j$





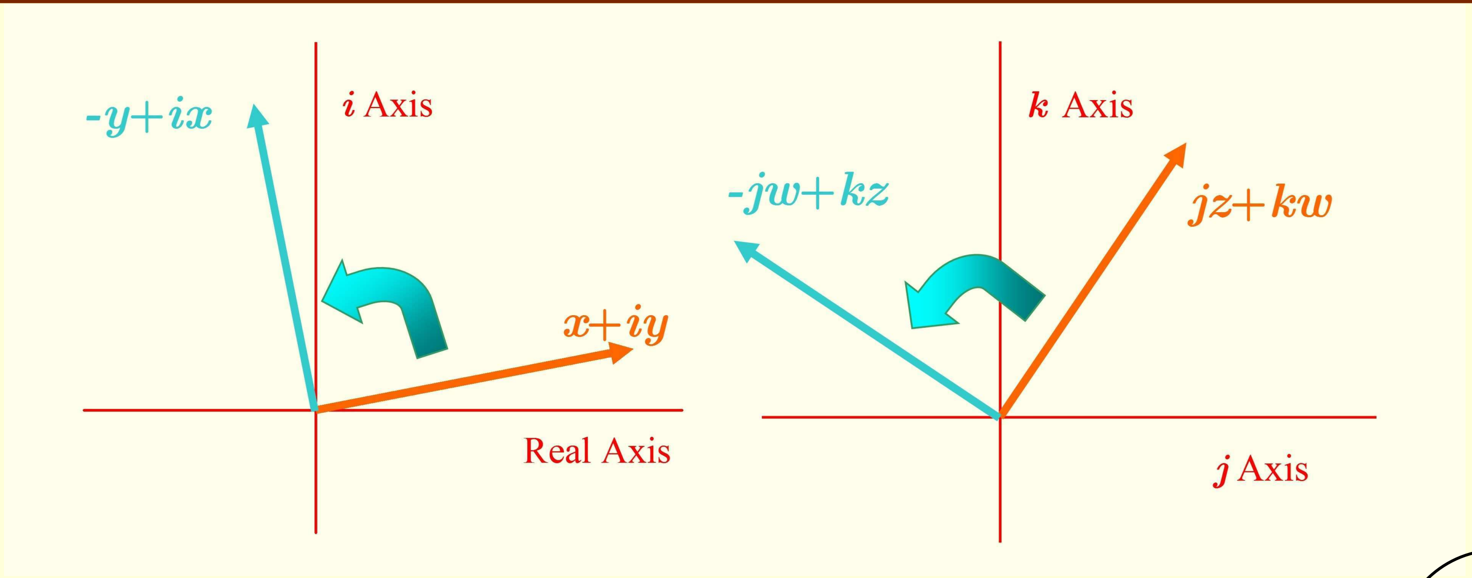
$$q = (x + y\,i) + (z + w\,i)\,j = x + y\,i + z\,j + w\,k$$

$$\boxed{ij = k = -ji;\, i^2 = j^2 = k^2 = -1}$$

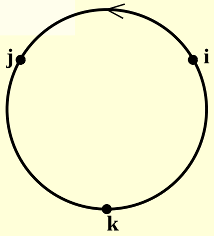
$\mathbb{H}$ is for Hamilton! ($\mathbb{Q}$ denotes rationals)

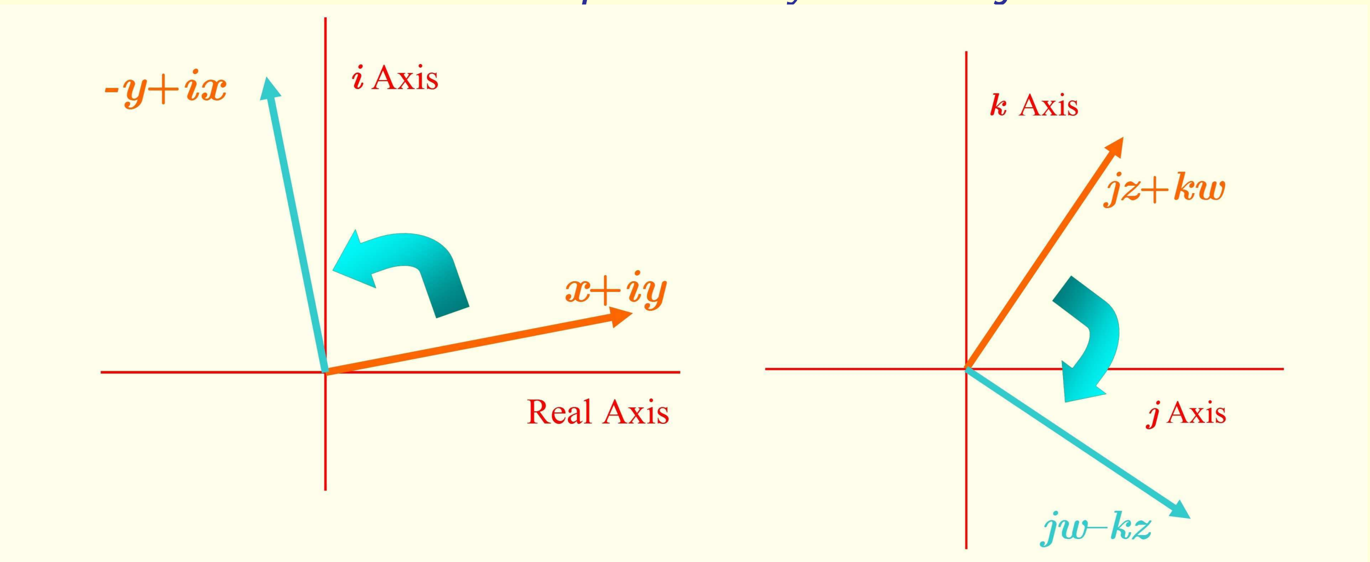
Calculate with your group: *iq* and *qi*

iq vs. qi



$$
\begin{aligned}
q &= x + iy + jz + kw \\
iq &= ix - y + kz - jw \\
qi &= ix - y - kz + jw
\end{aligned}
$$





Small Group Activity

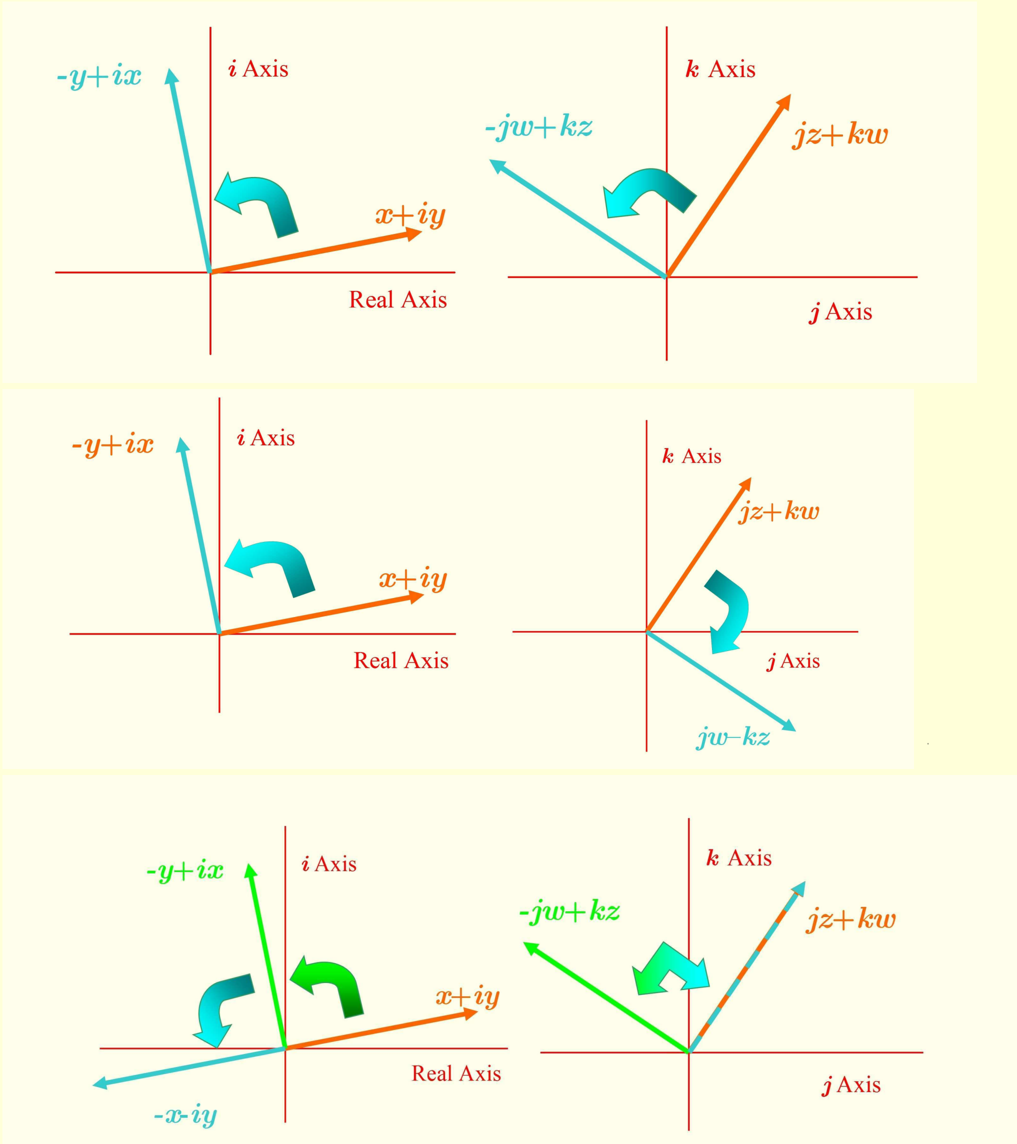
Things to consider:

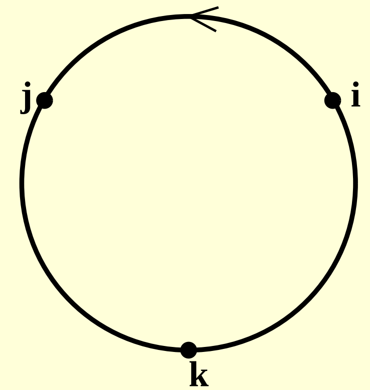
- Can emphasize more complex problems/reasoning.
- Student practice problem solving themselves.
- Equity: moves office hours into the classroom.

Classroom implementation:

- *You have 10 min., GO!*
- *Who needs help?*
- *Do you need more time?*
- *Pause.*

Conjugation



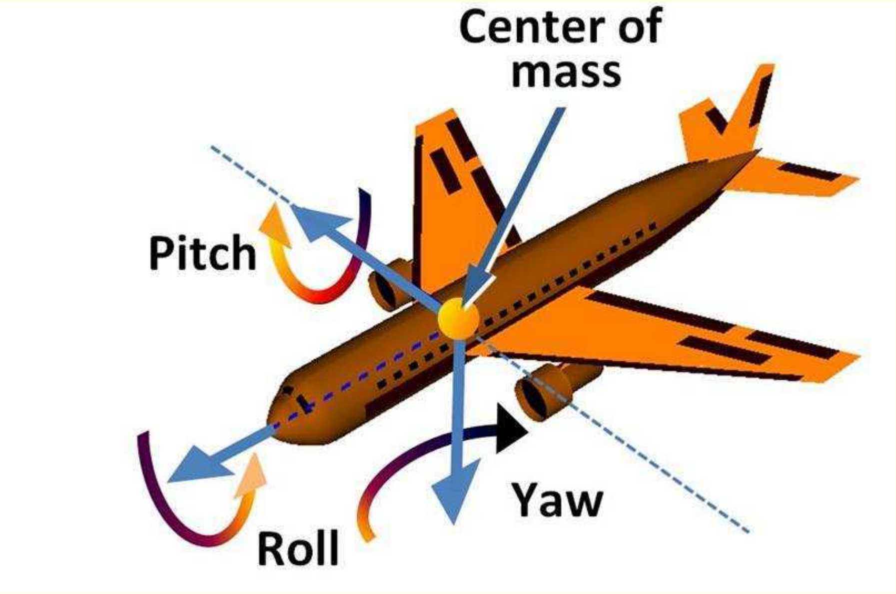


$$q = x + iy + jz + kw$$

$$iq = ix - y + kz - jw$$
$$qi = ix - y - kz + jw$$

$$iqi = -x - iy + jz + kw$$
$$-iqi = x + iy - jz - kw$$
(rotation in jk-plane)

Rotations



$$\boxed{q \longmapsto e^{i\theta/2} q e^{-i\theta/2}}$$

- $1 \longmapsto 1$; $i \longmapsto i$
- Rotates by θ "about i" (in jk-plane)
- $q \longmapsto e^{j\theta/2} q e^{-j\theta/2}$ rotates about j, etc.

$\therefore$ SO(3), the rigid rotations in 3 dimensions

Lecture (vs. Activities)

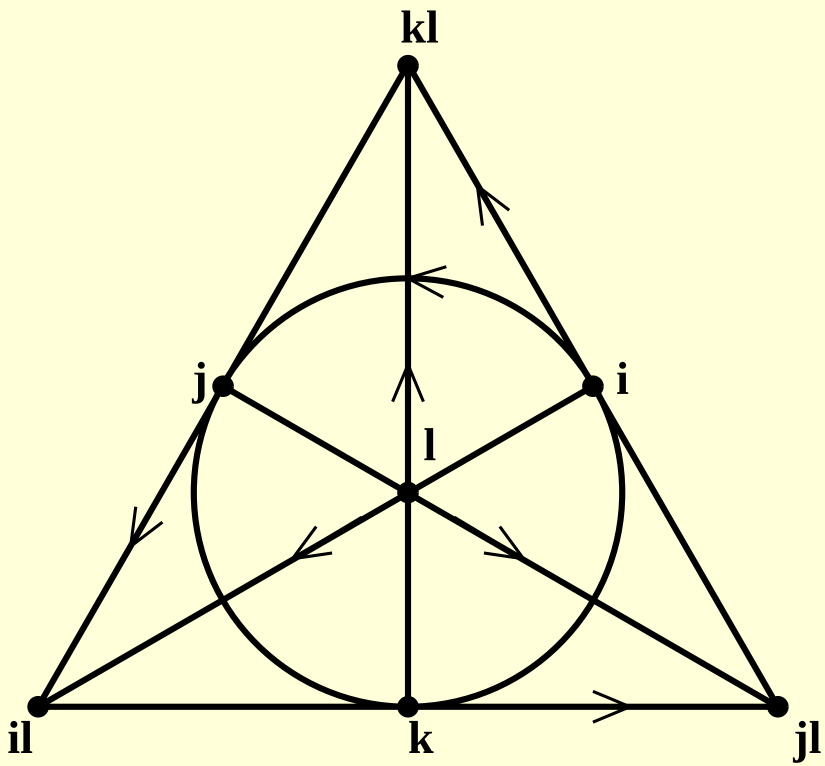
The Instructor:

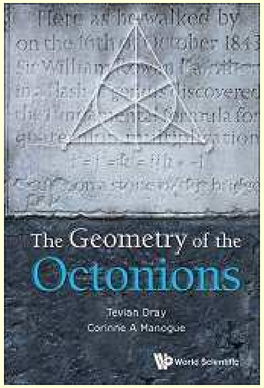
- Paints big picture.
- Inspires.
- Covers lots fast.
- Models speaking.
- Models problem-solving.
- Controls questions.
- Makes connections.
- Demonstrates new complicated reasoning.

The Students:

- Focus on subtleties.
- Experience delight.
- Slow, but in depth.
- Practice speaking.
- Practice problem-solving.
- Control questions.
- Make connections.
- Discover questions about what is complicated.

Generalizations





2015

Octonions! $(\mathbb{O} = \mathbb{H} + \mathbb{H}\ell)$

Use to model particle physics

http://octonions.geometryof.org/GO

Story Telling

Plum Muffins

Story telling is memorable.

Please return:

Please clean up your toys:

- Erase your whiteboard.
- Return the ABCD cards, whiteboard, and pen.