Mastering Partial Derivatives

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& The Paradigms in Physics Team
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Oregon State University
PER Physics Education Research
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- Utah State University

Small White Board Questions (SWBQs)

- Write something that you know about derivatives.

What is a Concept Image?

- Concept Image: the total cognitive structure that is associated with a concept, which includes all the mental pictures and associated properties and processes.

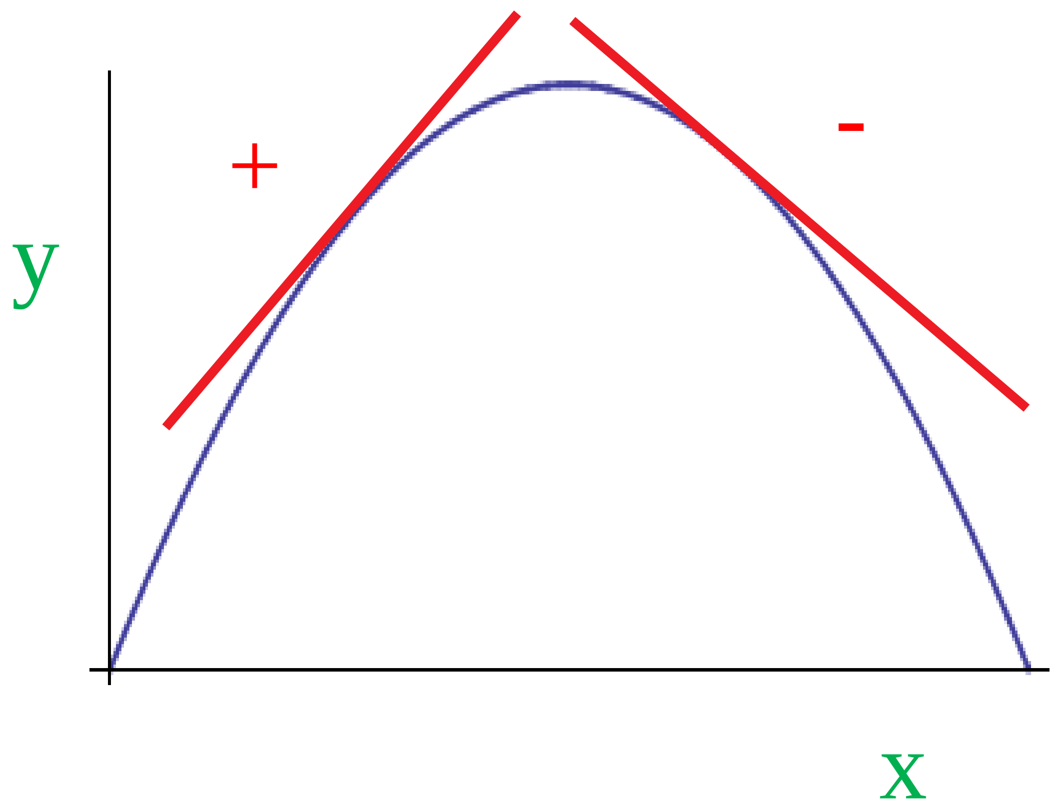
Tall and Vinner, *Educ. Stud. Math.*, (1981).

Concept Image of Derivative

- Ratio
- Slope
- Limit
- Function
- Rate of Change
- Velocity
- Difference Quotient

Lower Anchor for Derivatives

$$y = f(x)$$



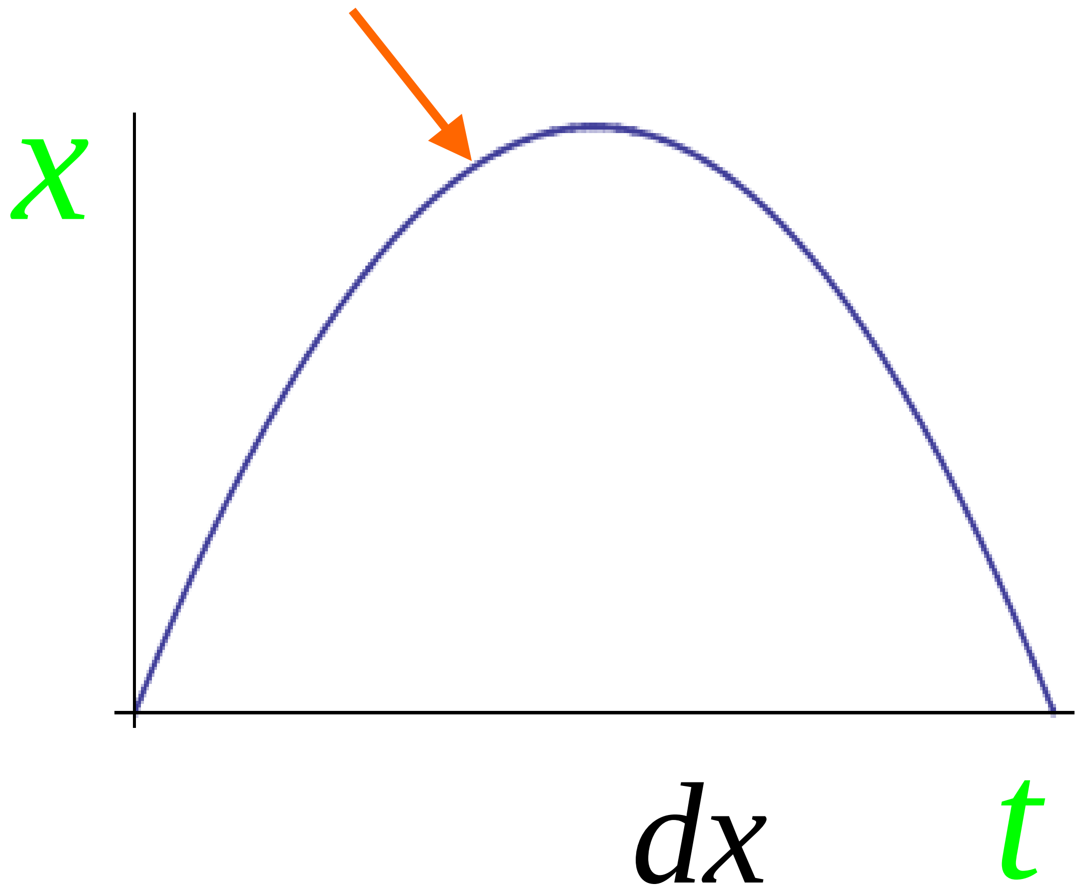
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$(\cos x)' = -\sin x$$

Derivative is slope of tangent line.

Mechanics—Lower Division

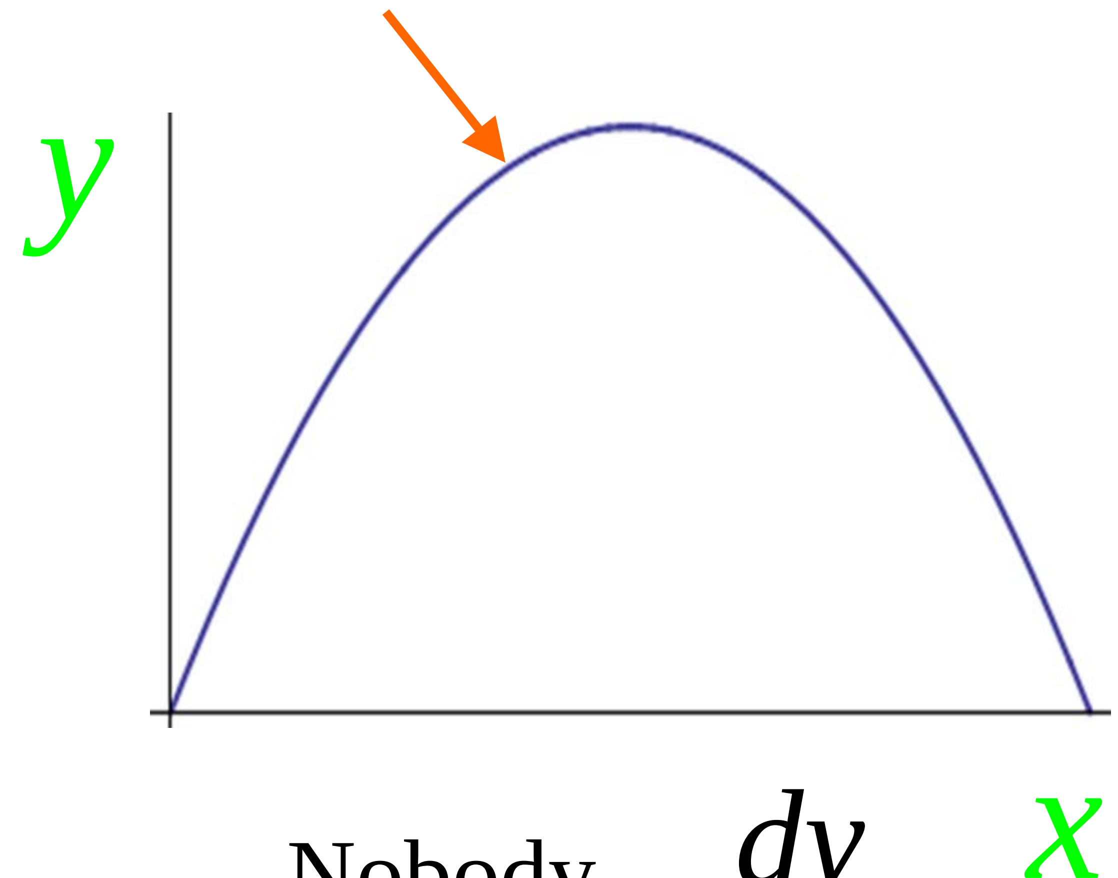
This is a Function



$$v = \frac{dx}{dt}$$

Derivative = Speed=Slope

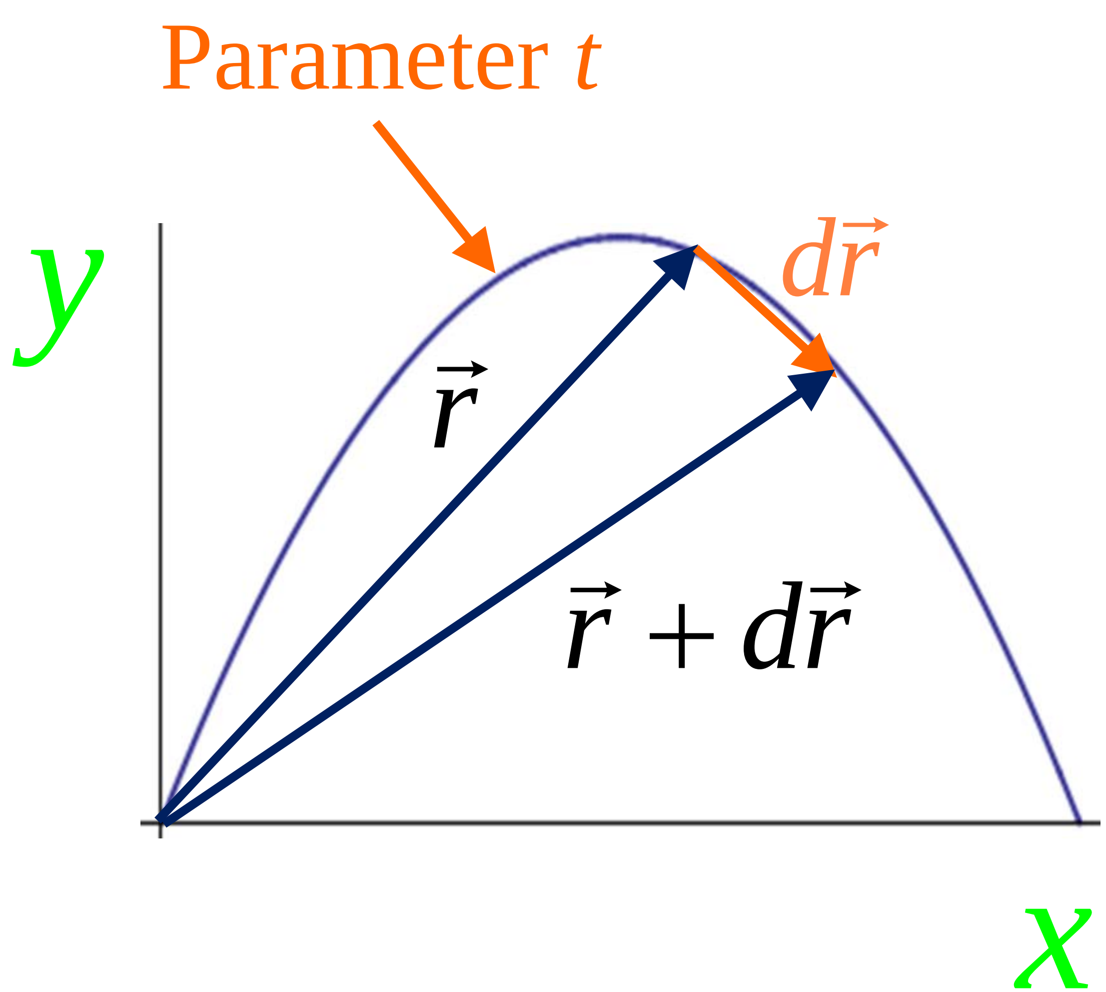
This is a Trajectory



$$\text{Nobody cares} = \frac{dy}{dx}$$

Derivative = Slope

Mechanics—Upper Division



Trajectory

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$= \frac{dx}{dt}\hat{x} + \frac{dy}{dt}\hat{y}$$

- Speed is NOT slope.
- Velocity points in direction of slope.

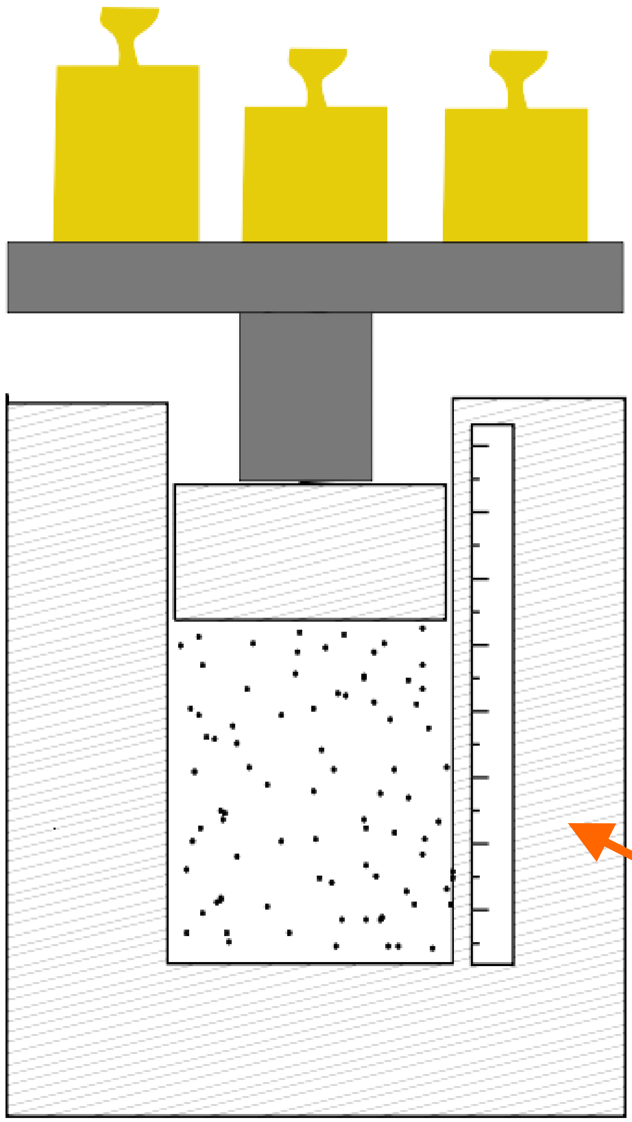
Name the Experiment

- Design an experiment to measure compressibility:

$$\beta_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \quad vs. \quad \beta_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S$$

Isothermal Isentropic

Name the Experiment

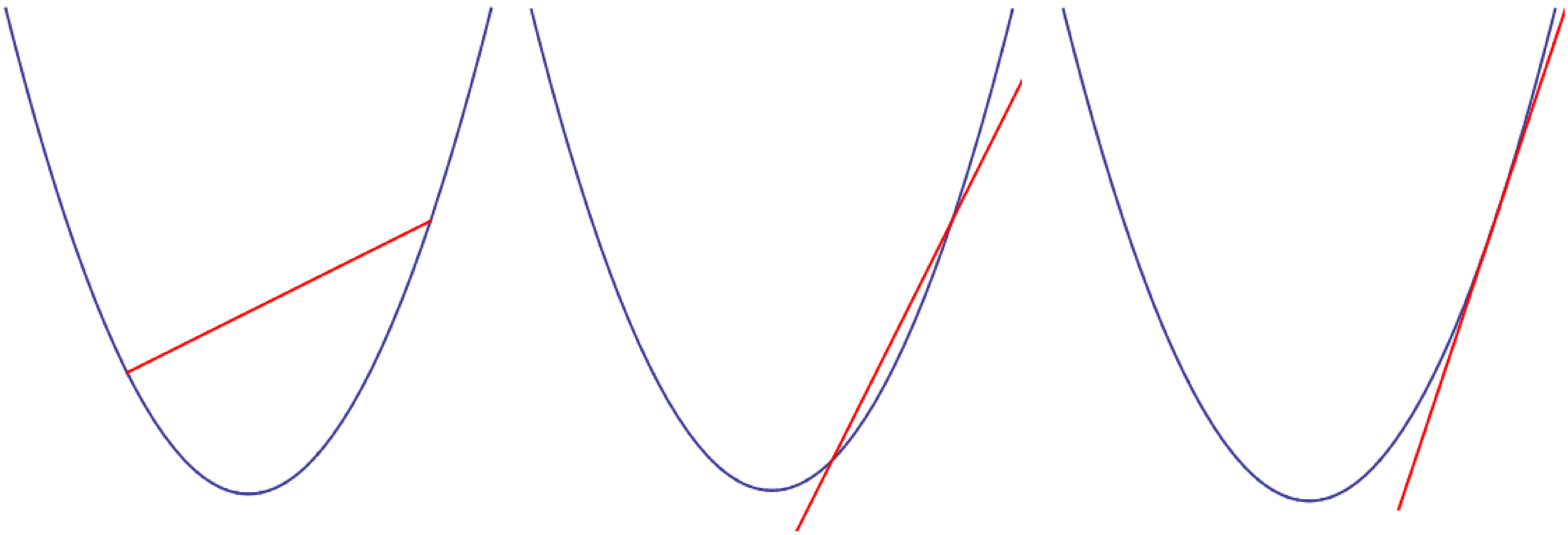


$$-\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \quad vs. \quad -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S$$

What is this material?

Linear Regime vs. Strict Limit

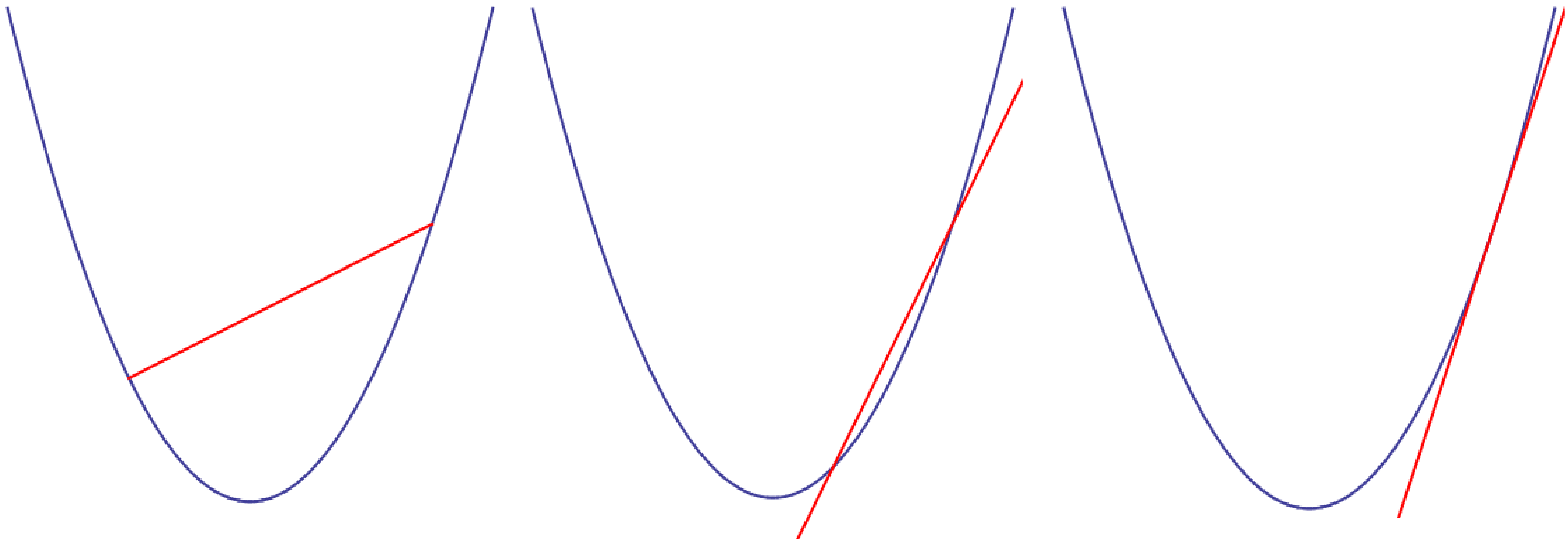
- Which diagram(s) represent the derivative?



- average vs. approximation vs. exact

Thick Derivatives

- What counts as a derivative?
  - Mathematicians: bright line at strict derivative.
  - Physicists: bright line at “good enough.”



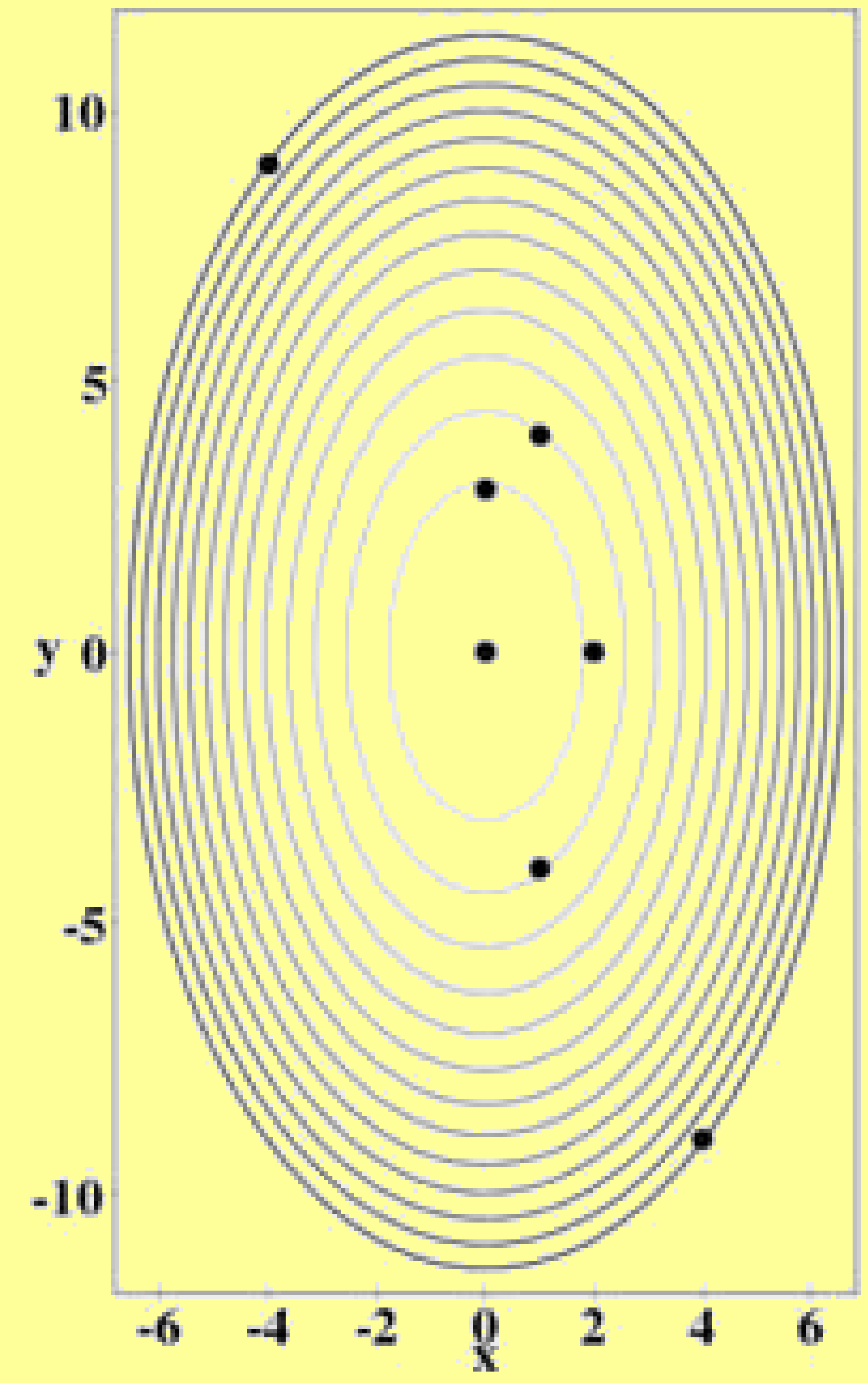
Concept Image of Gradient

- Use SWBQs to help students link elements of their concept image:

On your small white board, write ONE element of your concept image of gradient.

Kinesthetic Activity: Gradient

- Points in the direction of steepest change.
- Magnitude is slope.



Gradient: Which Direction?

Notations for Partial Derivatives

- Math vs Physics

$$f_x \equiv \frac{\partial f}{\partial x}$$

- Mechanics

$$\vec{f} = f_x \hat{x} + f_y \hat{y}$$

- E & M

$$E_x = -\left(\frac{\partial V}{\partial x} \right)$$

Equations Encode Meaning

$$grad\ f = \langle f_x, f_y, f_z \rangle$$

$$\vec{\nabla} V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$$

The Master Formula

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

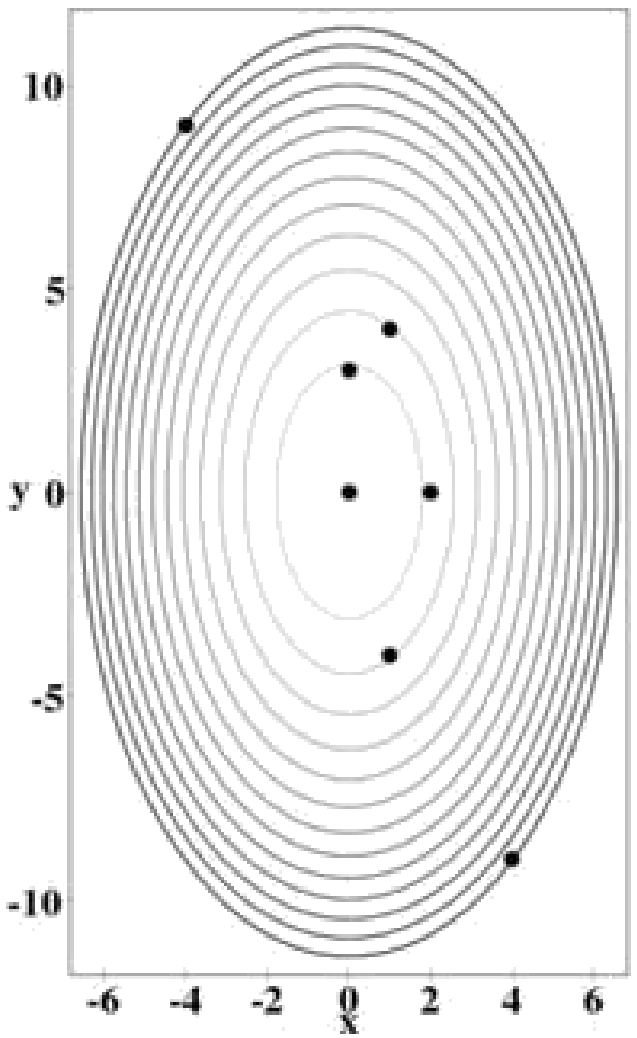
$$= \vec{\nabla} V \cdot d\vec{r}$$

$$\frac{dV}{ds} = \vec{\nabla} V \cdot \frac{d\vec{r}}{\left|d\vec{r}\right|} = \vec{\nabla} V \cdot \hat{T}$$

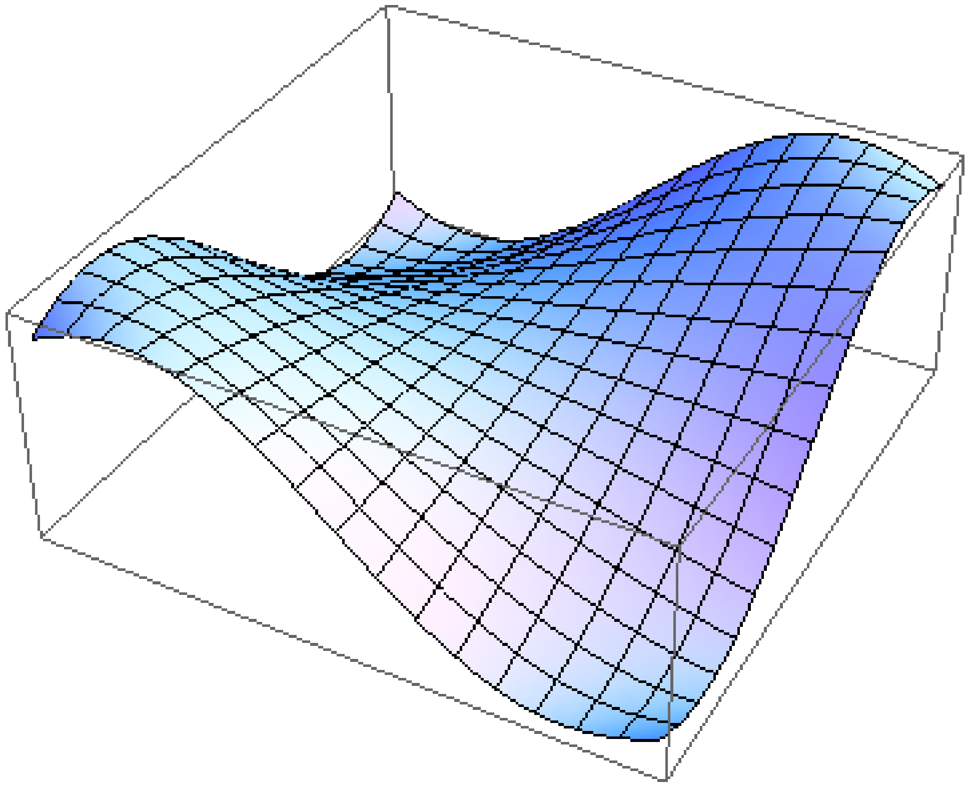
Representations

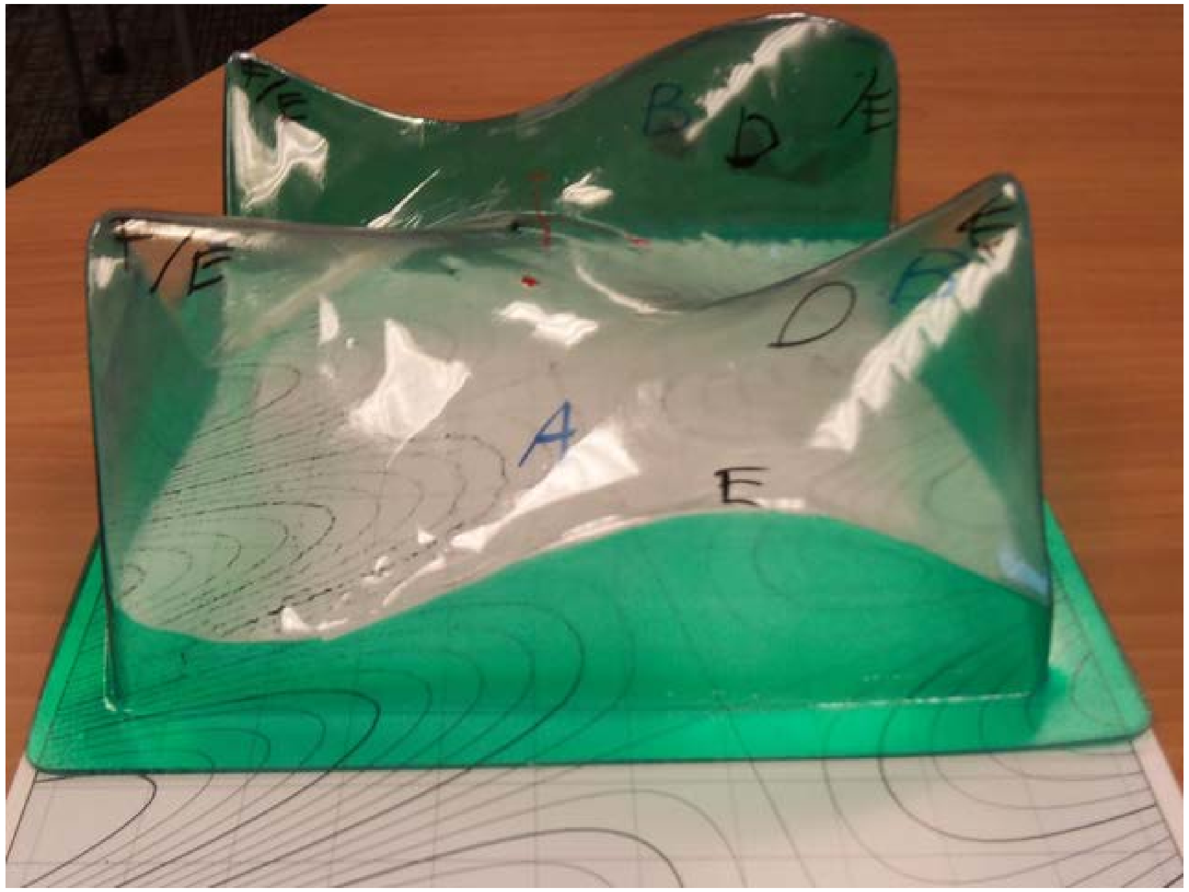
- How do we use activities with particular representations to scaffold connections between concept image elements?
  - Example: SWBQ encourages class discussion about multiple representations.
  - Example: Hill-pointing connects contour map with physical space.

Math Representations



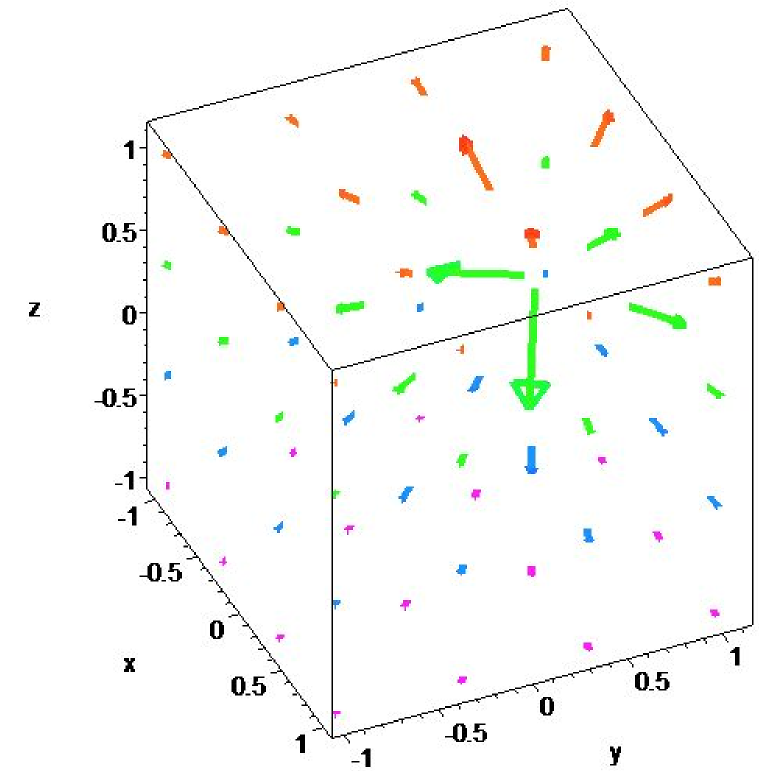
- Functions of 2 variables

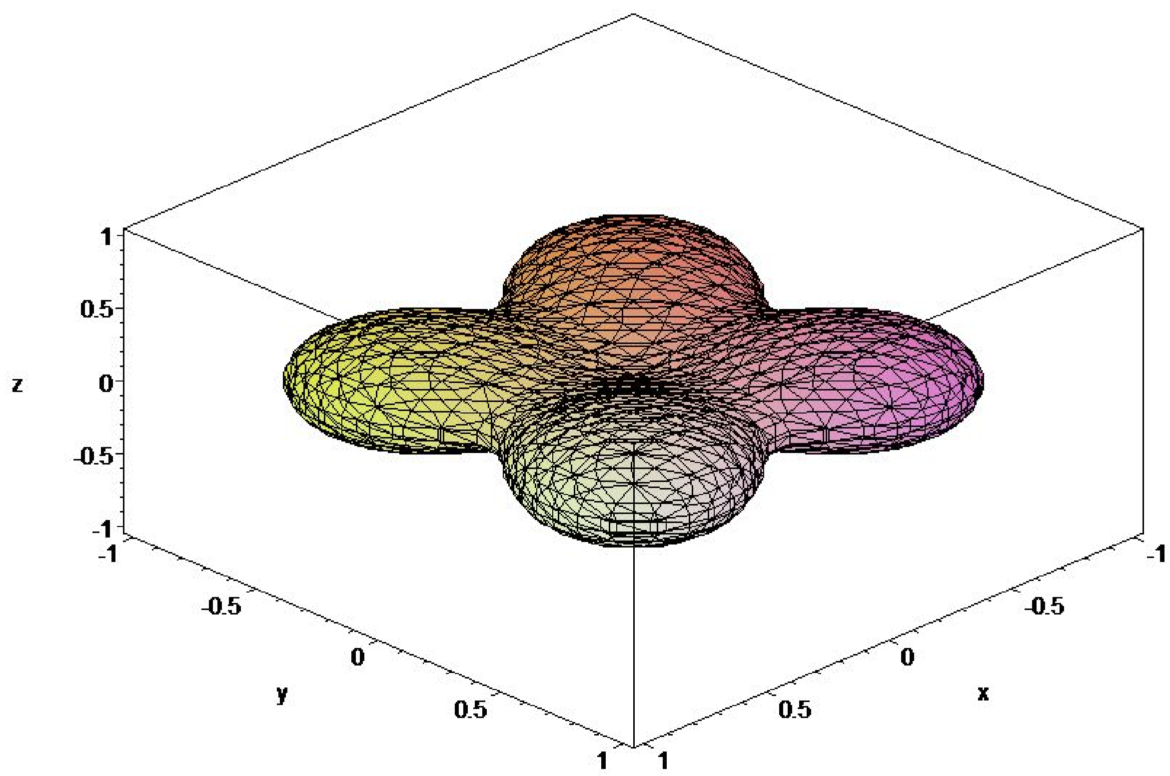


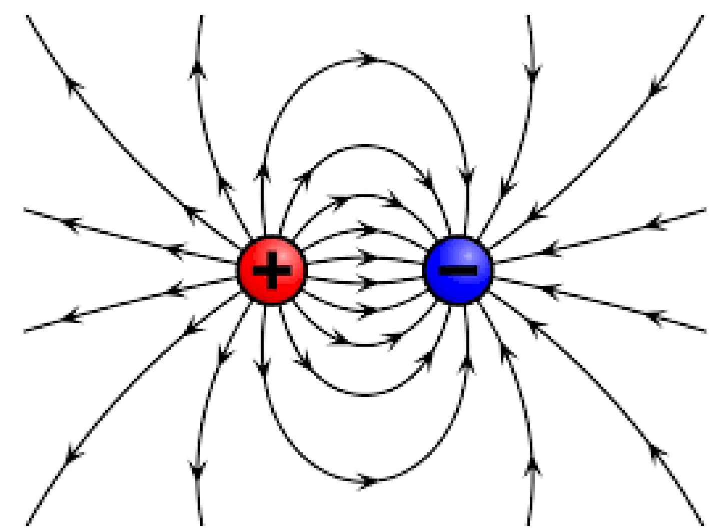


Physics Representations

- Functions of 3 variables
  - Equipotential Surfaces
  - 3-D Gradient Vectors
  - Electric Field Lines







Adopting Curriculum

- Old: Textbook authors determined order. lecture, reading, homework
- Now: Who determines the order?

  in-class activities, SWBQs/concept tests, mini-lectures, video, online short readings, flipping and backflipping, …

Conclusion

- The concept image of partial derivative has MANY, many, *many* elements!
- Experts use MANY representations.
- Different representations cue reasoning about different elements.
- Different subfields of mathematics and physics rely on different elements.
- Choose activities that foster connections between elements.

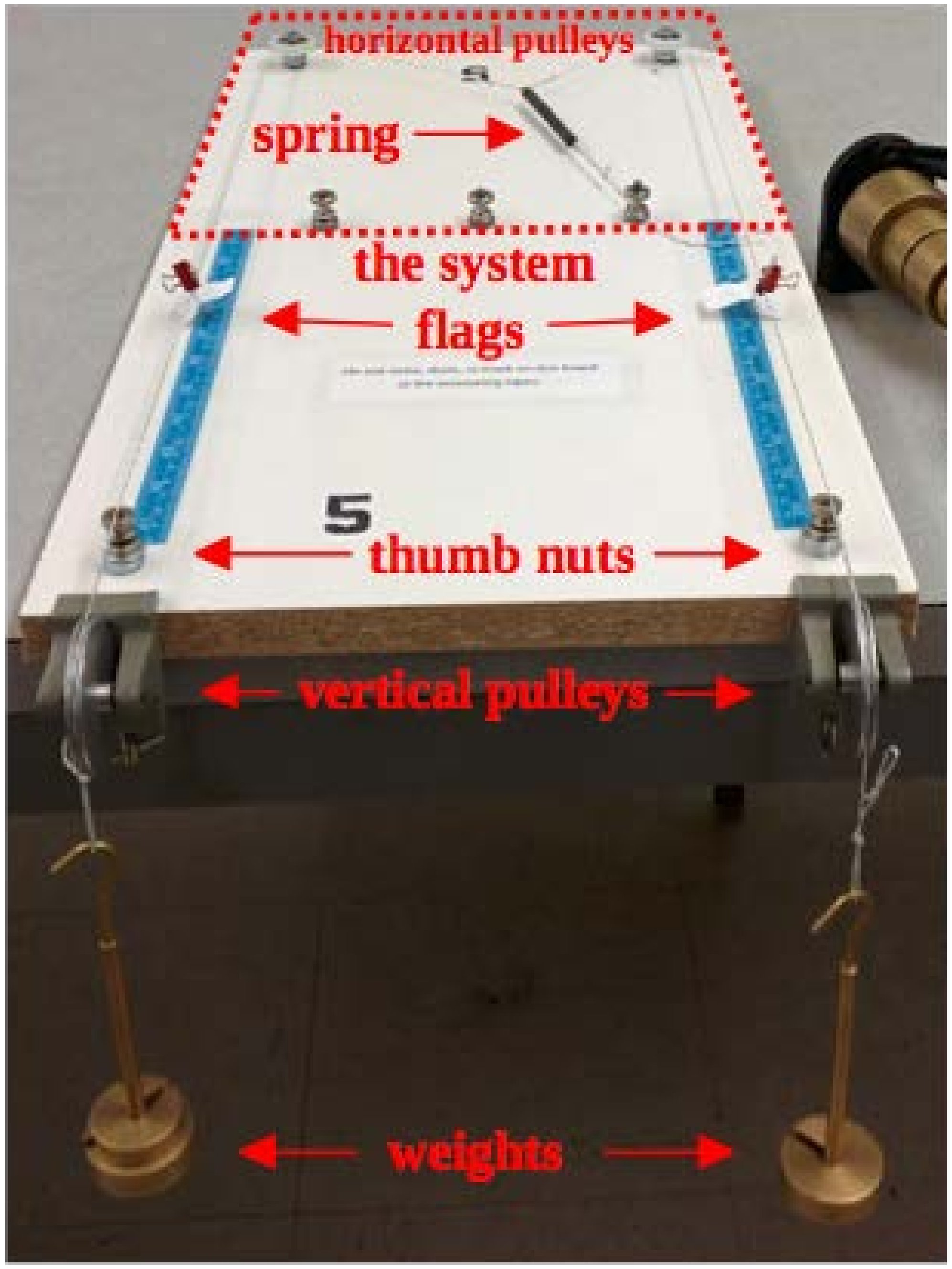
Research on Partial Derivatives

- What information can be easily extracted from particular representations?
- How do students change from one representations to another?
- What does expert problem solving look like?

Partial Derivatives Machine

David Roundy

Mike Vignal



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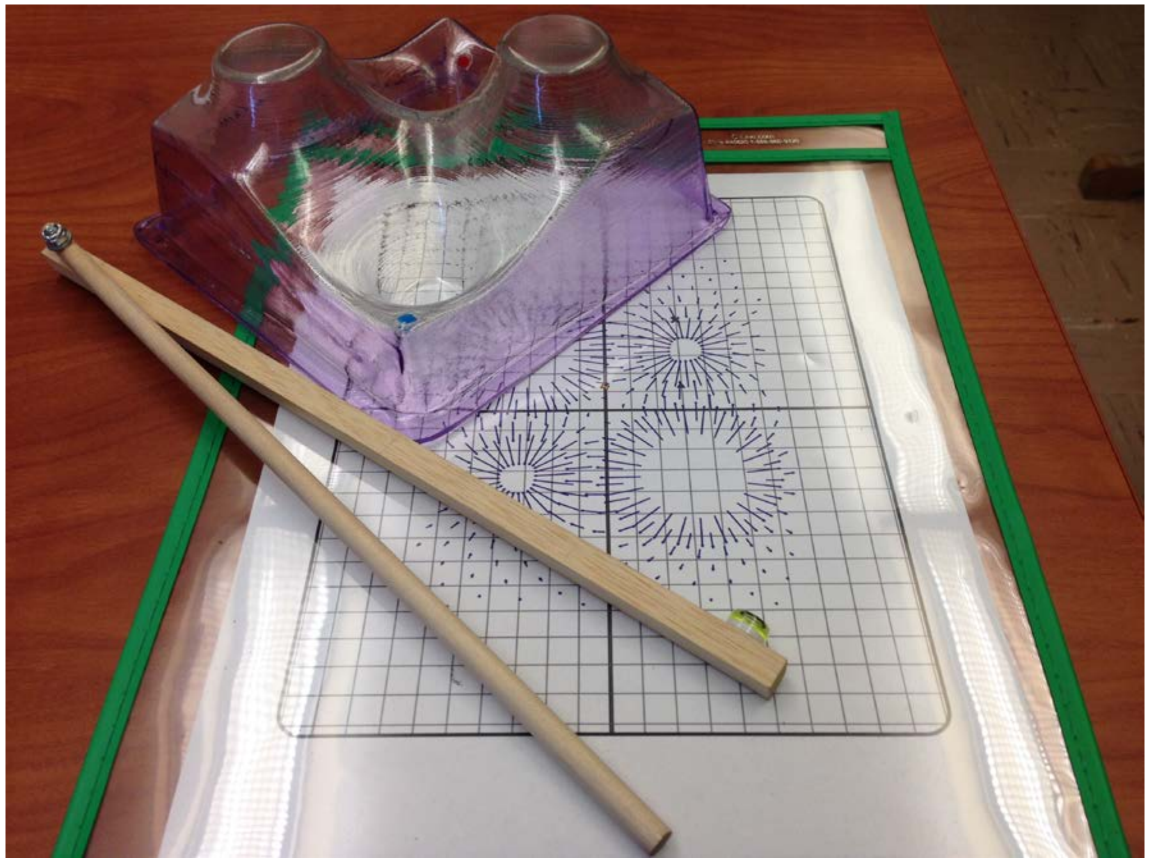
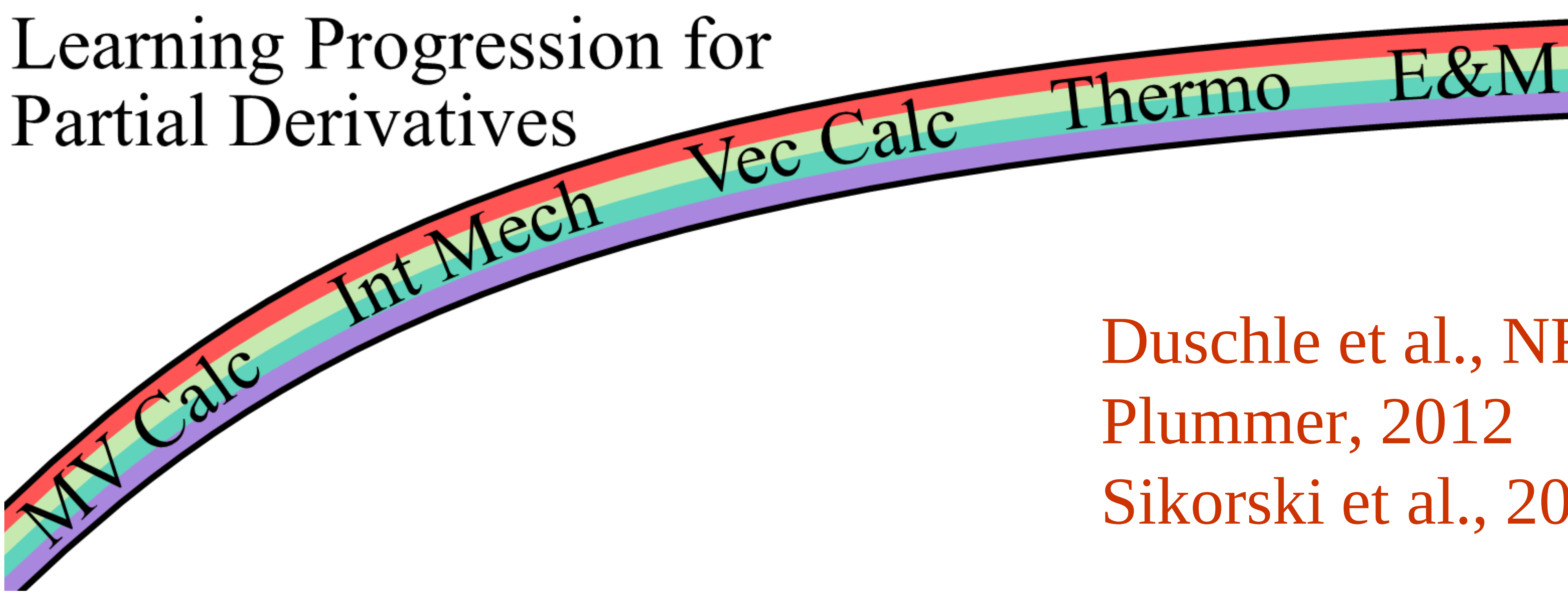
“Raising” Surfaces/Contour Mats



Elizabeth Gire
Aaron Wangberg
Robyn Wangberg

Learning Progressions

- Successively more sophisticated ways of thinking about a topic.
- Sequences that are supported by research on learner's ideas and skills.



Duschle et al., NRC, 2007
Plummer, 2012
Sikorski et al., 2009, 2010

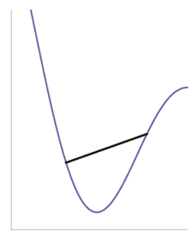
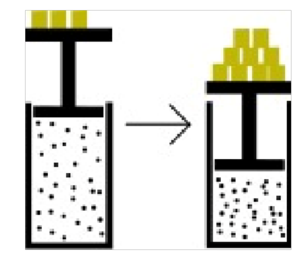
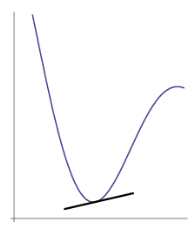
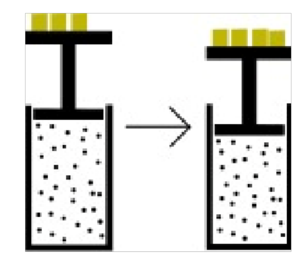
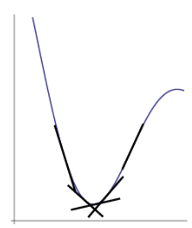
Learning Progressions

- What is an effective content sequence?
- Different types of resources: activities, SWBQs, text bits, homework problems, …
- What research supports these choices?

Learning Progressions

- Lower anchor grounded in prior ideas and skills students bring to the classroom.
- Upper anchor grounded in knowledge and practices of experts.

Extended Framework

Process-object layer	Graphical	Verbal	Symbolic	Numerical	Physical
	Slope	Rate of Change	Difference Quotient	Ratio of Changes	Measurement
Ratio		"avg. rate of change"	$\frac{f(x+\Delta x)-f(x)}{\Delta x}$	$\frac{y_2-y_1}{x_2-x_1}$ numerically	
Limit		"inst. rate of change"	$\lim_{\Delta x \to 0} \cdots$	...with Δx small	
Function		"...at any point/time"	$f'(x) =$ ...	... depends on x	tedious repeti-tion

Process-object layer	·	·	Symbolic	·	·
	Instrumental Understanding				
Function	*rules to "take a derivative"*				

Zandieh, CBMS Issues in Math Ed, 2000.

Roundy, et al., RUME, 2015.