The Geometry of Special Relativity

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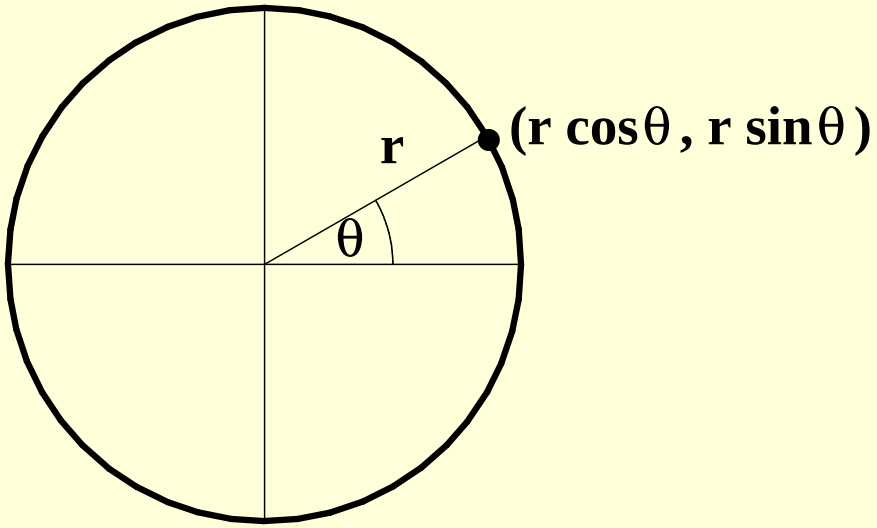
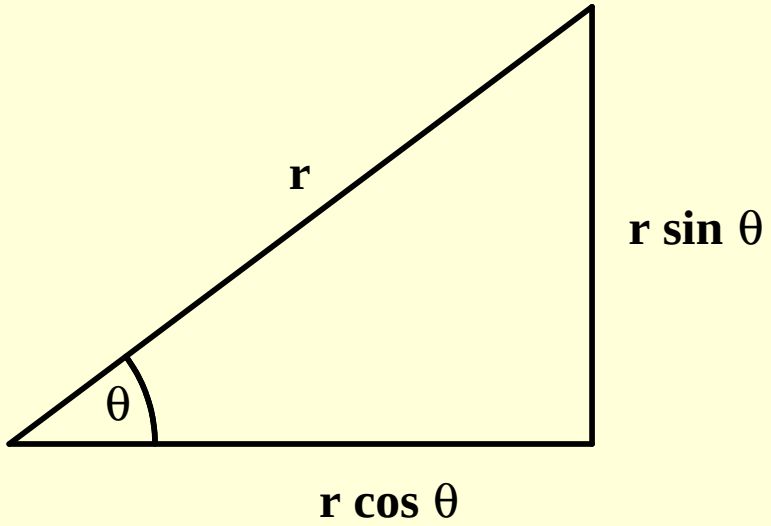
Circle Geometry

Write down something you know about trigonometry.

Circle Geometry



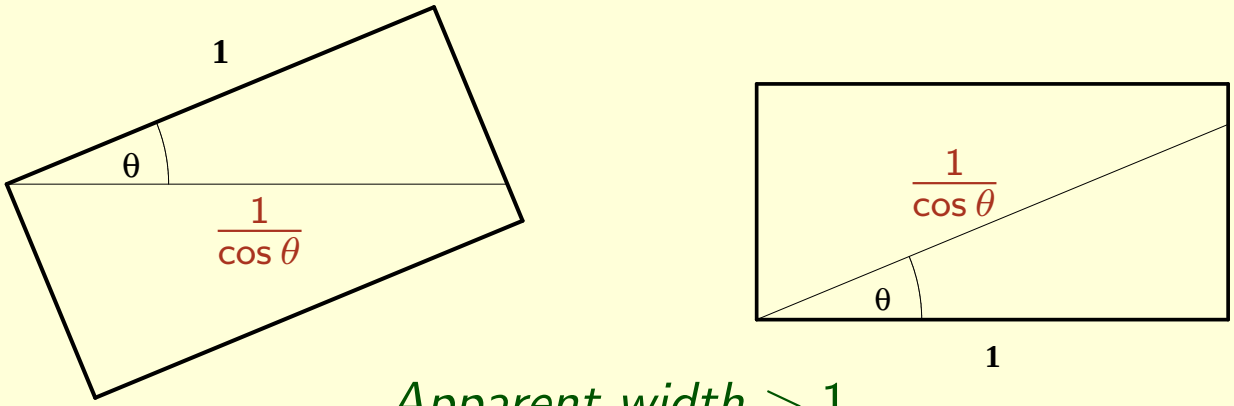


$r\theta =$ arclength

If $\tan\theta = \frac{3}{4}$, what is $\cos\theta$?

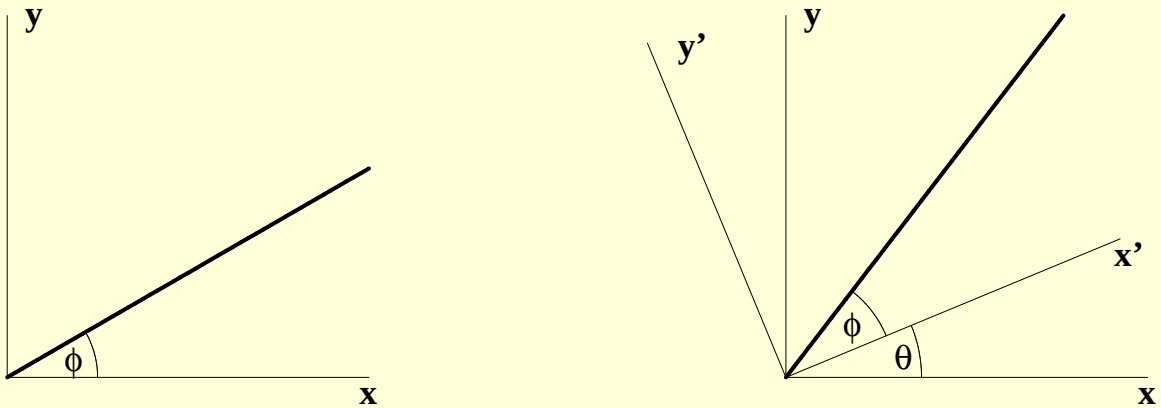
Measurements

Width:



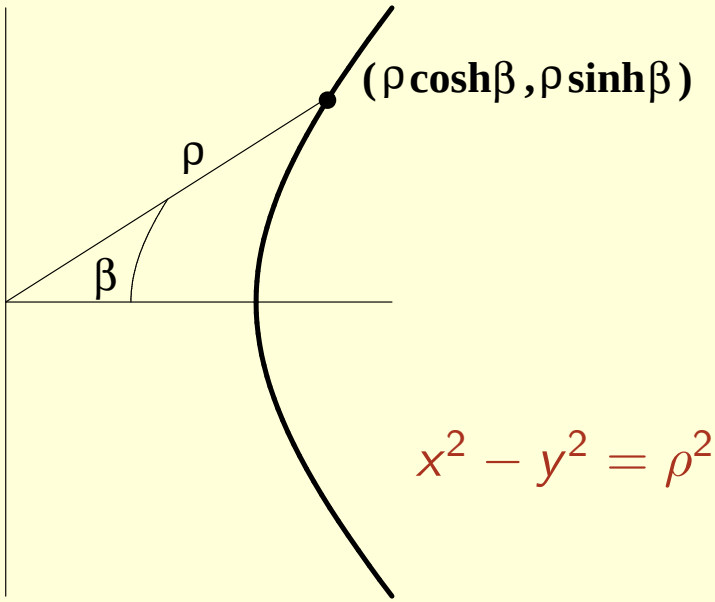
Apparent width > 1

Slope:



$$m = \tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = \frac{m_1 + m_2}{1 - m_1 m_2}$$

Hyperbola Geometry



$$x^2 - y^2 = \rho^2$$

$$\rho\beta = \text{arclength}$$

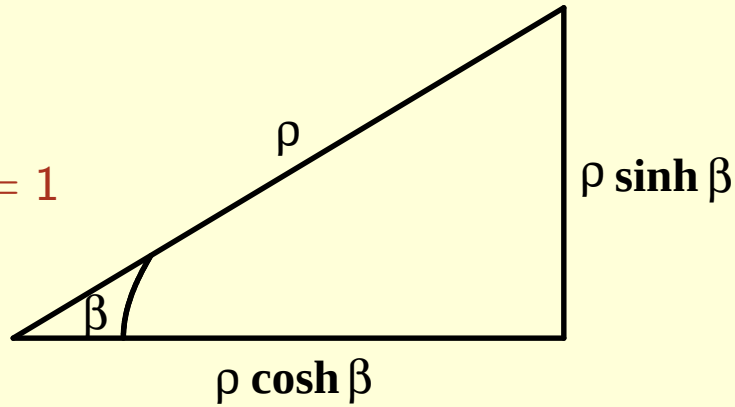
$$ds^2 = |dx^2 - dy^2|$$

$$\cosh\beta = \frac{1}{2}\left(e^{\beta} + e^{-\beta}\right)$$

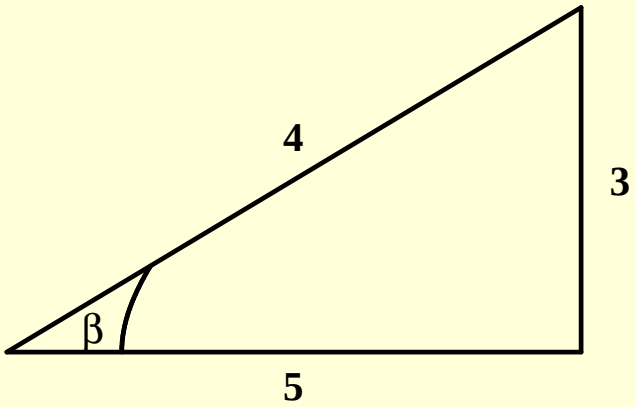
$$\sinh\beta = \frac{1}{2}\left(e^{\beta} - e^{-\beta}\right)$$

Hyperbolic Triangle Trig

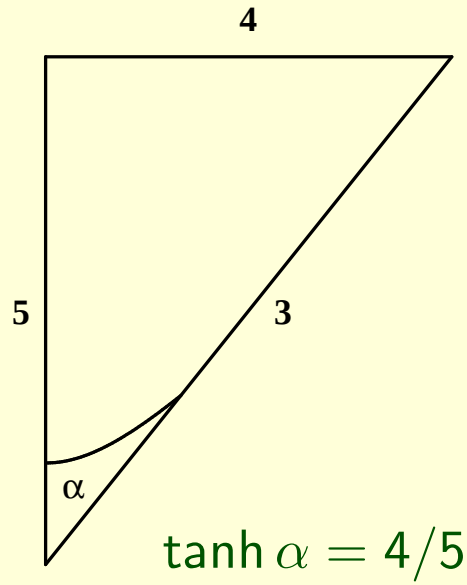
$\cosh^2 \beta - \sinh^2 \beta = 1$



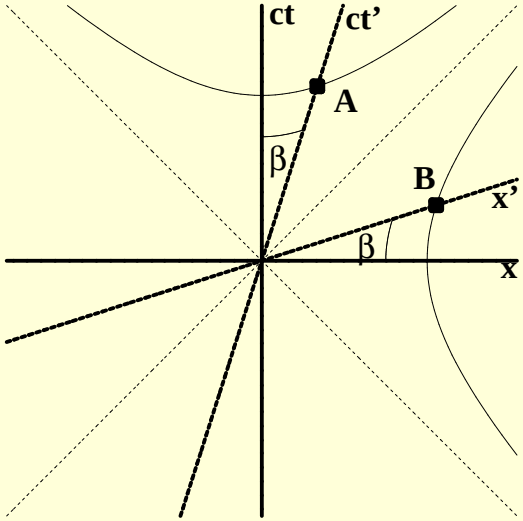
Draw a 3–4–5 triangle in hyperbola geometry.



$\tanh \beta = 3/5$

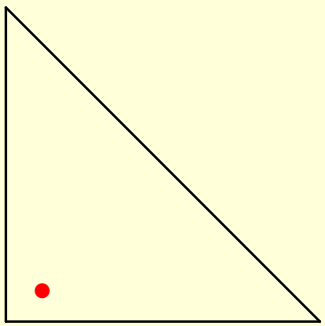
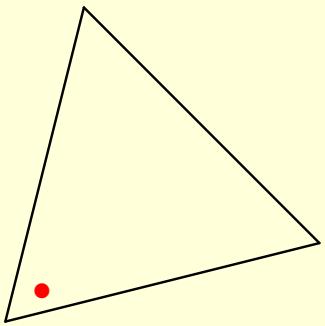
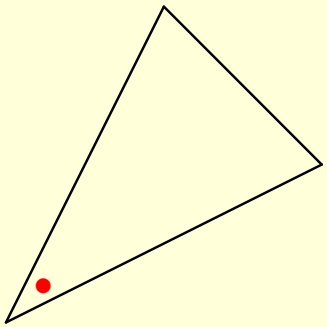


$\tanh \alpha = 4/5$



$$x^2 - ct^2 = x'^2 - ct'^2$$
$$(c = 1)$$

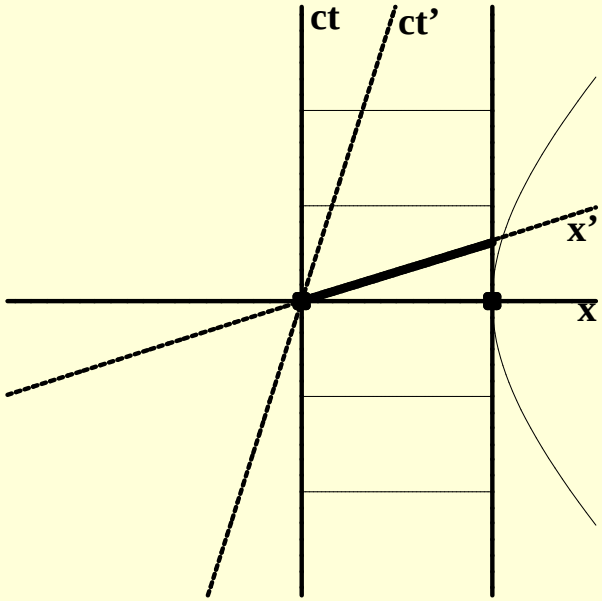
Draw a right triangle in hyperbola geometry.

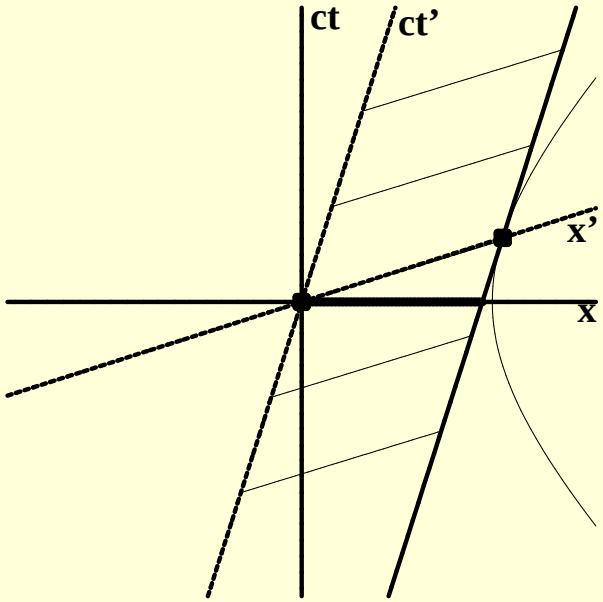




"right angles" are not angles!

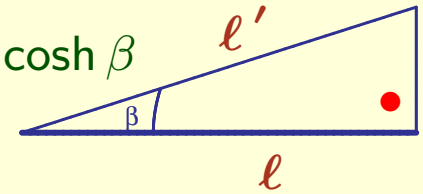
Length Contraction

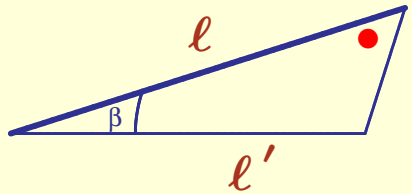
Draw a spacetime diagram showing a meter stick at rest.





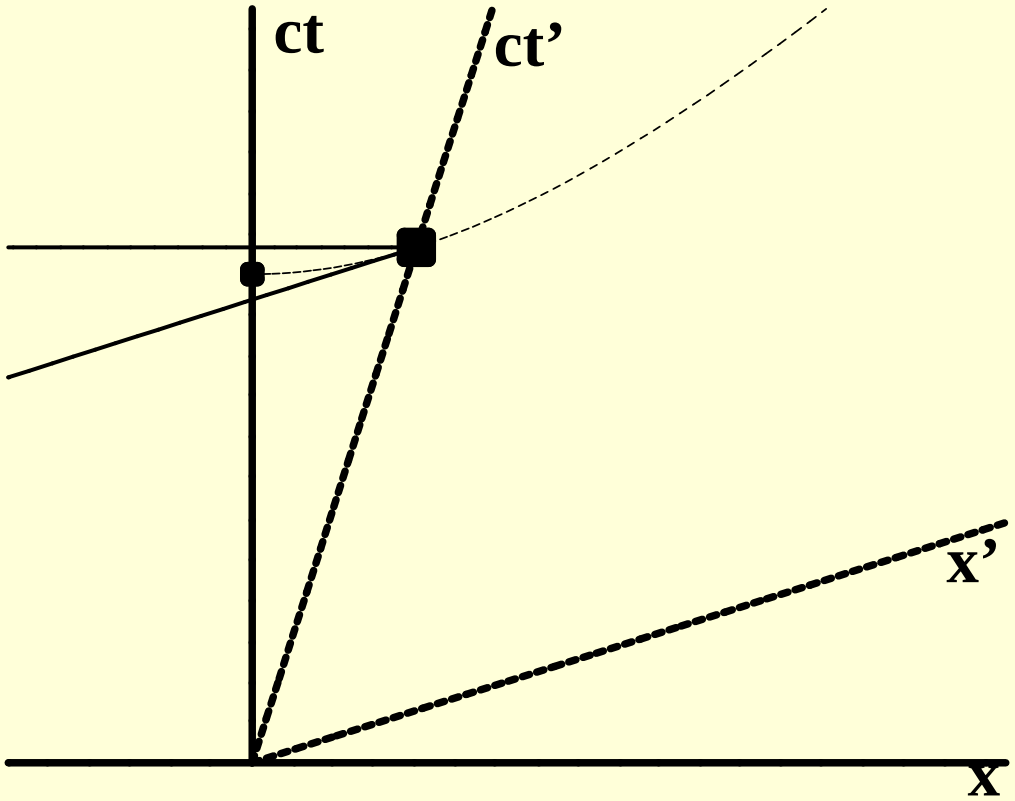
$\ell = \ell' \cosh\beta$





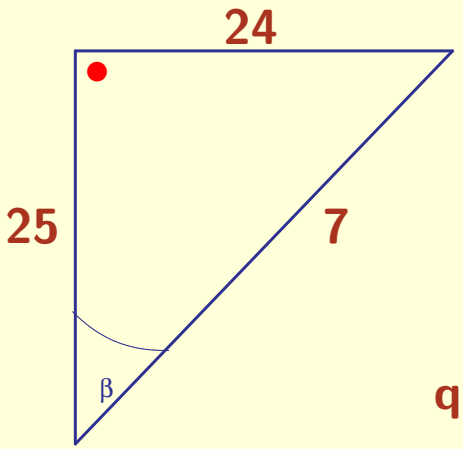
$$\ell' = \frac{\ell}{\cosh\beta}$$

Time Dilation

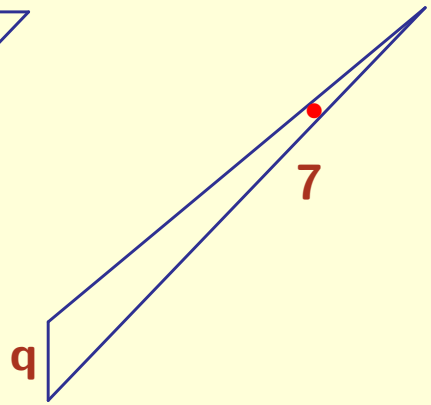


Twin Paradox

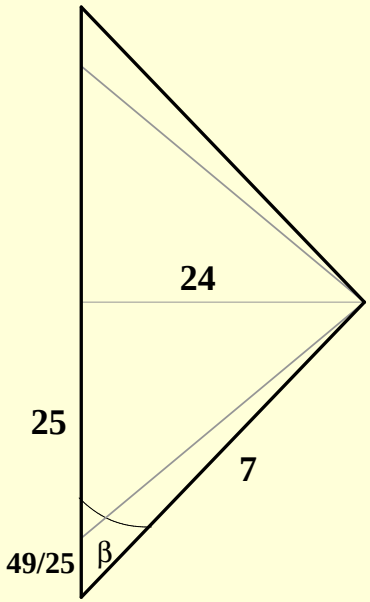
One twin travels 24 light-years to star X at speed $\frac{24}{25}c$; her twin brother stays home. When the traveling twin gets to star X, she immediately turns around, and returns at the same speed. How long does each twin think the trip took?



$$\cosh\beta = \frac{25}{7}$$

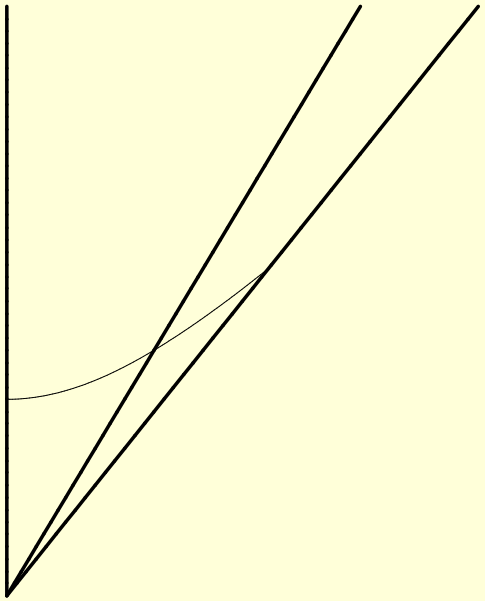


$$q = \frac{7}{\cosh\beta} = \frac{49}{25}$$



Straight path takes longest!

Addition of Velocities



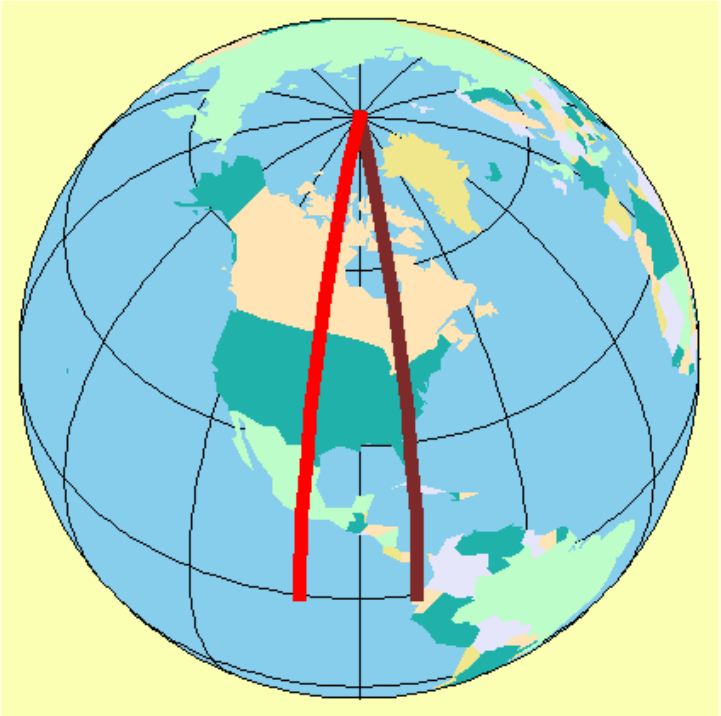
$$\frac{v}{c} = \tanh\beta$$

$$\tanh(\alpha+\beta) = \frac{\tanh\alpha + \tanh\beta}{1+\tanh\alpha\tanh\beta} = \frac{\frac{u}{c}+\frac{v}{c}}{1+\frac{uv}{c^2}}$$

Einstein addition formula!

Which Geometry?

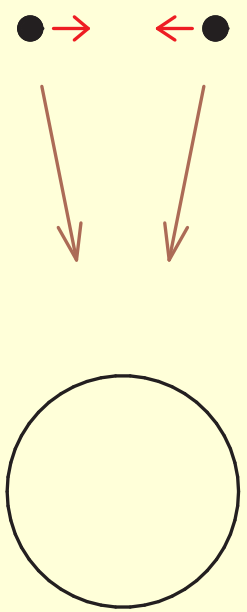
signature	flat	curved
$(+ + \ldots +)$	Euclidean	Riemannian
$(- + \ldots +)$	Minkowskian	



Tidal forces!

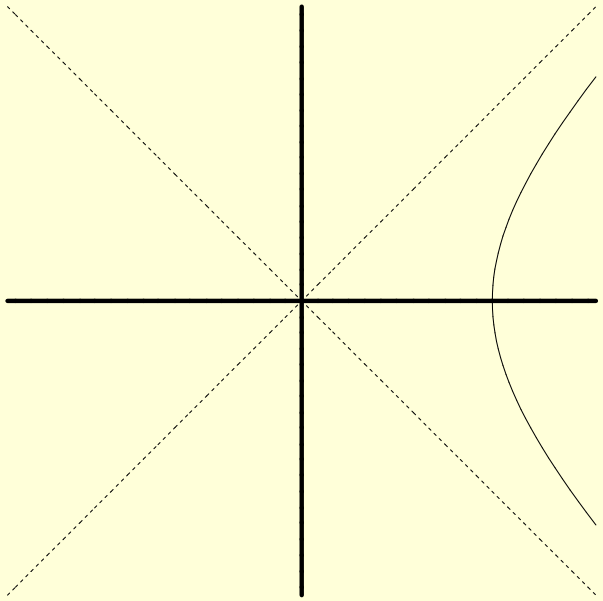
Which Geometry?

signature	flat	curved
$(+ + \ldots +)$	Euclidean	Riemannian
$(- + \ldots +)$	Minkowskian	Lorentzian



General Relativity!

Einstein: gravity=acceleration



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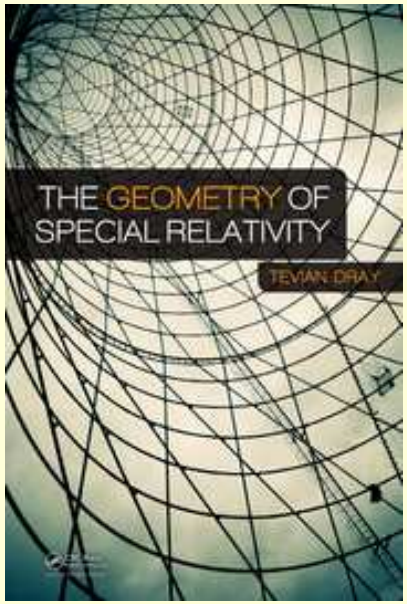

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