Problem 1: Be warned: this is a tough one that will test your algebra skills, but it’s pretty cool. A *catenary* is the shape of a chain or cable hanging from two fixed endpoints, and in general is given by

\[ a \cosh \left( \frac{x}{a} \right) = a \left( \frac{e^{x/a} + e^{-x/a}}{2} \right) \]

Let’s just let \( a = 1 \) to make our lives easier, so \( f(x) = \cosh(x) \). Find the arc-length of the catenary on the interval \([0,1]\). You can do it!

Side note: *catenary* and *hyperbolic trig functions* make fascinating reading.

Problem 2: (i) A 20m chain with a mass-density of 3kg/m is coiled on the ground. How much work is performed lifting the chain so that it is fully extended (and one end touches the ground)?

(ii) How much work is performed to lift 1/4 of the chain?

Problem 3: Indiana Jones is in a pickle. The switch to open a door is the side panel (which measures 4m \( \times \) 4m) of an empty aquarium. Exactly 18\( \rho g \) Newtons of force are required to activate the panel; even 1N too much or too little and spikes fall from the ceiling killing everyone in the room. Find how high Indy should fill the tank with water (with density \( \rho \) and acceleration due to gravity \( g \)) to activate the switch safely.

Problem 4: An inverted conical tank is 3m tall and 1m in diameter at its widest point. The water is being pumped out of a spout 2m above the top of the tank. Recall that the density of water is \( \rho = 1000\text{kg/m}^3 \).

(i) Find the work needed to empty the tank if it is full. Include units.

(ii) Find the work to empty half the tank (assuming it is full to begin with).