Department of Mathematics OSU
Qualifying Examination
Fall 2015

PART II: COMPLEX ANALYSIS and LINEAR ALGEBRA

- Do any two of the three problems in Part CA, *use the correspondingly marked blue book* and indicate on the selection sheet with your identification number those problems that you wish graded. Similarly do any two of the three problems in Part LA.

- Your solutions should contain all mathematical details. Please write them up as clearly as possible.

- Explicitly state any standard theorems, including hypotheses, that are necessary to justify your reasoning.

- On problems with multiple parts, individual parts may be weighted differently in grading.

- You have three hours to complete Part II.

- When you are done with the examination, place examination blue books and selection sheets back into the envelope in which the test materials came. You will hand in all materials. If you use extra examination books, be sure to place your code number on them.

PART CA : COMPLEX ANALYSIS QUALIFYING EXAM

1. Let f be analytic on the unit disc D, and assume that $|f(z)| < 1$ on D. Prove that if f fixes two distinct points in D, then $f(z) = z$.

2. Fix a real number $\lambda > 1$. Let

$$h(z) = \exp(-z) + z - \lambda.$$

Prove that $h(z)$ has exactly one root in the open half-plane defined by $\Re(z) > 0$.

3. Use complex path integrals over closed curves and the theory of residues to calculate the following improper definite integrals, with rigorous justification of all steps.

(a) $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{(x^2+1)^3} dx$

(b) $\displaystyle\int_{-\infty}^{+\infty} \frac{\cos x}{x^2+1} dx$

Exam continues on next page ...

PART LA: LINEAR ALGEBRA QUALIFYING EXAM

1. Let T be a symmetric linear operator on $\mathbb{R}^n$. Prove that if $\mathrm{tr}(\mathrm{T}^2) = 0$, then $T = 0$.

2. Suppose A and M are $n \times n$ matrices over $\mathbb{C}$. A is invertible and $AMA^{-1} = M^2$. Prove that all nonzero eigenvalues of M are roots of unity.

3. Let $M_n(\mathbb{R})$ denote the set of real $n \times n$ matrices, and define

$$S := \{A \in M_n(\mathbb{R}) \mid A \text{ orthogonal, with all eigenvalues } \neq -1\}$$
$$T := \{B \in M_n(\mathbb{R}) \mid B^t = -B\}.$$

Prove that the map $F(A) = (I - A)(I + A)^{-1}$ defines a bijection from S to T.