Department of Mathematics OSU
Qualifying Examination
Fall 2014

PART II: COMPLEX ANALYSIS and LINEAR ALGEBRA

- Do any **two** of the problems in Part CA, *use the correspondingly marked blue book* and indicate on the selection sheet with your identification number those problems that you wish graded.

  Similarly for Part LA.

- Your solutions should contain all mathematical details. Please write them up as clearly as possible.

- Explicitly state any standard theorems, including hypotheses, that are necessary to justify your reasoning.

- On problems with multiple parts, individual parts may be weighted differently in grading.

- You have three hours to complete Part II.

- When you are done with the examination, place examination blue books and selection sheets back into the envelope in which the test materials came. You will hand in all materials. If you use extra examination books, be sure to place your code number on them.

PART CA: COMPLEX ANALYSIS QUALIFYING EXAM

1. Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ and let $f : \mathbb{D} \to \mathbb{C}$ be a nonconstant holomorphic function. Suppose there exists $M > 0$ such that

$$\limsup_{n\to+\infty} |f(\alpha_n)| \le M,$$

whenever $\{\alpha_n\}$ is a sequence in $\mathbb{D}$ converging to a point on the boundary of $\mathbb{D}$. Prove that $|f(z)| < M$ for all $z \in \mathbb{D}$.

2. (a) Consider the curve $\gamma : [0, 2\pi] \to \mathbb{C}$ defined by $\gamma(t) = 2\cos t + i2\sin t$. Calculate

$$\int_\gamma \frac{1}{z^3 \cos z} dz.$$

(b) Use a complex path integral to calculate

$$\int_0^{2\pi} \frac{\cos(3\theta)}{\cos\theta} d\theta.$$

3. Represent the function

$$f(z) = \frac{z}{z^2+1}$$

(a) as a Laurent series valid in the annulus $0 < |z - i| < 2$.

(b) as a Laurent series valid in the annulus $|z - i| > 2$.

(c) What type of singularity does f have at i?

Exam continues on next page ...

PART LA: LINEAR ALGEBRA QUALIFYING EXAM

1. Let V be a finite dimensional complex inner product space.

   (a) Prove that if $T : V \to V$ is a linear transformation such that $\langle Tv, v \rangle = 0$ for all $v \in V$, then $T = 0$.

   (b) Does the result from (a) hold if we assume that V is a real inner product space? Prove or give a counterexample.

2. Prove, or supply a counterexample: If A is an invertible $n \times n$ complex matrix and some power of A is diagonalizable over $\mathbb{C}$, then A is diagonalizable over $\mathbb{C}$.

3. Let A be a real $n \times n$ matrix. A *real cube root* of A is a real $n \times n$ matrix B satisfying $B^3 = A$.

   (a) Show that if A is symmetric then it has a real cube root.

   (b) Find a real 3×3 matrix which does not have a real cube root (with proof).