YOUR PROJECT TITLE HERE

AUTHOR 1 AND AUTHOR 1

Advisor: Your Advisor's Name
Oregon State University

Abstract. This is an abstract of the work we are working on. Woo!

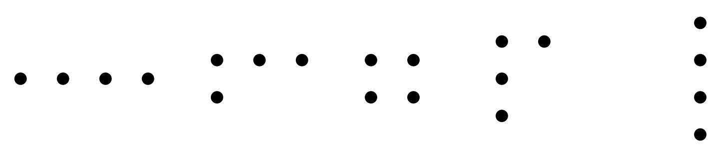
1. Introduction and Statement of Results

A *partition* of n is a non-increasing sequence of positive integers that sum to n, where each summand is called a *part*. The partition function $p(n)$ is defined to count the number of partitions of n. For example, we see that

$$4 = 3 + 1 = 2 + 2 = 2 + 1 + 1 = 1 + 1 + 1 + 1,$$

give all of the partitions of $n = 4$, so $p(4) = 5$. Partitions can be represented by Ferrers diagrams, for example, the following give the Ferrers diagrams of the partitions of 4.

$4 \quad 3+1 \quad 2+2 \quad 2+1+1 \quad 1+1+1+1$



Ramanujan discovered and proved the following congruences involving the function $p(n)$.

Theorem 1.1.

$$p(5n+4) \equiv 0 \pmod{5} \tag{1}$$

$$p(7n+5) \equiv 0 \pmod{7} \tag{2}$$

$$p(11n+6) \equiv 0 \pmod{11}. \tag{3}$$

The *rank* of a partition λ was defined by Dyson to be the largest part of λ, $l(\lambda)$, minus the number of parts, $n(\lambda)$. For example, if

$$\lambda = 5 + 4 + 4 + 4 + 2 + 1 + 1 + 1,$$

then we have

$$\text{Dyson rank}(\lambda) = 5 - 8 = -3.$$

Date: August 14, 2020.

This work was done during the Summer 2020 REU program in Mathematics and Theoretical Computer Science at Oregon State University supported by NSF grant DMS-1757995.

Throughout, we let $N(s,m,n)$ denote the number of partitions of n with rank congruent to s modulo m.

In response to a conjecture of Dyson, Atkin and Swinnerton-Dyer [2] found a number of elegant formulas, in terms of modular functions and generalized Lambert series, for the generating functions for rank differences of the form

$$N(r,\ell,\ell n+d)-N(s,\ell,\ell n+d)$$

for the cases $\ell = 5$ and $\ell = 7$. Their work showed that the rank provides a combinatorial explanation for the celebrated Ramanujan congruences given in (1) and (2). Note that this doesn't prove all of the congruences given in Theorem 1.1, only the first two.

References

[1] George E. Andrews. *The theory of partitions*. Addison-Wesley Publishing Co., Reading, Mass.-London-Amsterdam, 1976. Encyclopedia of Mathematics and its Applications, Vol. 2.

[2] AOL Atkin and P Swinnerton-Dyer. Some properties of partitions. *Proceedings of the London Mathematical Society*, 3(1):84–106, 1954.

First College
E-mail address: email1@FC.edu

Second University
E-mail address: email2@SU.edu