ON THE ZEROS OF SOME FAMILIES OF MODULAR FUNCTIONS

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Abstract. For genus 0 subgroups $\Gamma \subset PSL_2(\mathbb{R})$, the function field of the modular curve $X(\Gamma)$ is generated by a single element, the Hauptmodul. When $\Gamma$ contains the congruence subgroup $\Gamma_0(N)$ for some $N \in \mathbb{Z}$, the Hauptmodul may sometimes be associated to the largest sporadic simple group, the so-called “Monster,” through the coefficients of its Fourier series, an example of a moonshine correspondence. This relationship gives rise to a “twisted Hecke action” $\hat{T}_n$ on a Hauptmodul $f$ such that

$$\{n(f|\hat{T}_n) \mid n \in \mathbb{N}\} = \{q^n + O(q) \mid n \in \mathbb{N}\}$$

is a basis for the space of modular functions for $\Gamma$. For the $j$-invariant of $PSL_2(\mathbb{Z})$, the twisted Hecke action agrees with the standard Hecke operators on modular forms, and Asai, Kaneko, and Ninomiya (1998) demonstrated the associated basis of modular functions have all of their zeros on the unit circle in the standard fundamental domain for $PSL_2(\mathbb{Z})$. We describe an extension of their method to the normalizer of $\Gamma_0(N)$ for some small $N$. 

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